## Thermal behaviour of a multilayer media in transient regime

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#### Abstract

In this paper, we develop a mathematical model, which calculate the temperature and thermal contact resistance distributions in a multilayer media. This work is composed of two parts. The first part concerns the analytical solution of the conduction thermal problem for two plates, which ended in mathematical expressions giving the temperatures and thermal resistance profiles. $A$ computer code, which calculates the analytical expressions from the values of the temperature at any time as well as the thermal contact résistance, is elaborated in the second part. Résumé - Dans cet article, nous développons un modèle mathématique, qui calcule la température et les distributions des résistances de contact thermiques dans des milieux multicouche. Ce travail se compose de deux parties. La première partie concerne la solution analytique du problème thermique de conduction pour deux plans, qui se terminent en expressions mathématiques donnant les températures et les profils des résistances thermiques. Un code machine, qui calcule les expressions analytiques des valeurs de la température à tout moment aussi bien que le résistance de contact thermique, est élaboré dans la deuxième partie.


Keywords: Temperature - Thermal contact resistance - Thermal conduction - Transient state Multilayer media.

## 1. INTRODUCTION

The problems of one-dimensional, heat transfer in a plate have numerous applications: heat removal from a plate heat exchanger element of a thermal installation by the coolant fluid, heat dissipation from a current-carrying plate, etc. In the case of multilayer media, the conduction heat transfer takes place through two layers, having different thermal conductivity, and with perfect thermal contact between these layers (Fig. 1). Unfortunately, this thermal contact is not, in general, perfect.

Several experimental and theoretical works have reported in the literature on the prediction of thermal contact résistance [1-5]. For example, in the work of Yeh et al. [1], an experimental study of thermal contact conductance was conducted with pairs of aluminium alloy (6061-T6) specimens jointed by bolts. Results show that the interfacial contact pressure increases with an increase of either the applied torque or the number of bolts. An experimental investigation was carried out to study the behaviour of thermal contact resistance (TCR) at the interface of metallic double tubes with respect to governing parameters by Bourouga and Bardon [2]. The results show that, on a set of samples of the similar kind, the TCR presents a minimum value with increasing assembly pressure and temperature level. Monte [3] investigated the transient heat conduction problems in one-dimensional multi-layer solids are usually solved applying conventional techniques, based on Vodicka's approach and the separation-of-variables method. Zhou [4] obtained an analytical solution for transient heat conduction in hollow cylinders containing wellstirred fluid with uniform heat sink. A two-dimensional (axisymmetric) transient heat conduction in components computer program (HCC) was successfully developed for predicting engine

[^0]combustion chamber wall temperatures, by Liu and Reitz [5]. The alternating direction explicit (ADE) Saul'yev method was used in their code.

Our approach appears rather simple to the means used, since it is necessary to make submit one of the faces an impulse of heat and to measure the temperature of the other face. The value of the thermal contact resistance is obtained from a computer code developed here, based on an analytical calculation, by comparing between the measured and calculated temperature.

## 2. PROBLEM FORMULATION

Consider two plates as illustrated in Fig. 1. The plates dimensions are sufficiently are supposed very large to the thickness, in order to ensure only the temperature gradient in the thickness direction $x$. Therefore, we can take: ' $a$ ', for the first plate, and ' $b$ ', for the second plate, ' $a$ ' and ' $b$ ' are the thickness of the first and the second plate respectively. These thicknesses are very small than other dimensions of the plates (Fig. 1).


Fig.1: Cross-section of two plates
We subject at $x=0$, an impulse of heat, type-flash, which will heat a small portion of the thickness $\varepsilon$ of the first layer [6].

## 3. MATHEMATICAL EQUATIONS

The temperatures distributions $\mathrm{T}_{1}(\mathrm{x})$ and $\mathrm{T}_{2}(\mathrm{x})$ in the first and second plate respectively, are governed by the following heat conduction equations:

$$
\begin{align*}
& \alpha_{1} \cdot \frac{\partial^{2} \mathrm{~T}_{1}}{\partial \mathrm{x}^{2}}=\frac{\partial \mathrm{T}_{1}}{\partial \mathrm{t}}  \tag{1}\\
& \alpha_{2} \cdot \frac{\partial^{2} \mathrm{~T}_{2}}{\partial \mathrm{x}^{2}}=\frac{\partial \mathrm{T}_{2}}{\partial \mathrm{t}} \tag{2}
\end{align*}
$$

Subject to the temperature boundary conditions, for $\mathrm{t}>0$ :

$$
\begin{array}{ll}
\frac{\partial \mathrm{T}_{1}}{\partial \mathrm{x}}=0 & \text { at } \mathrm{x}=0 \\
\lambda_{1} \frac{\partial \mathrm{~T}_{1}}{\partial \mathrm{x}}=\lambda_{2} \frac{\partial \mathrm{~T}_{2}}{\partial \mathrm{x}} & \text { at } \mathrm{x}=\mathrm{a}
\end{array}
$$

$\mathrm{T}_{1}=\mathrm{T}_{2}$
at $\quad \mathrm{x}=\mathrm{a}$
$\lambda_{2} \frac{\partial \mathrm{~T}_{2}}{\partial \mathrm{x}}+\mathrm{h}_{2} \mathrm{~T}_{2}=0$
at $\quad \mathrm{x}=\mathrm{a}+\mathrm{b}$
and the initial conditions, at $t=0$ :

$$
\begin{array}{ll}
\mathrm{T}_{1}=\mathrm{T}_{2}=\mathrm{T}_{0}=20^{\circ} \mathrm{C} & \text { at } \quad \varepsilon<\mathrm{x} \leq \mathrm{a}+\mathrm{b} \\
\mathrm{~T}_{1}=\mathrm{T}_{0}^{\prime}=\frac{\mathrm{Q}}{\rho \cdot \mathrm{C}_{\mathrm{p}} \cdot \varepsilon} & \text { at } \quad 0 \leq \mathrm{x} \leq \varepsilon \tag{8}
\end{array}
$$

In order to obtain the analytic solution of each equation (1) and (2), we use the solution by separation of variables [6-8]:

$$
\begin{equation*}
\mathrm{T}_{1}(\mathrm{x}, \mathrm{t})=\Psi_{1}(\mathrm{x}) \cdot \Gamma_{1}(\mathrm{t}) \tag{9}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{T}_{2}(\mathrm{x}, \mathrm{t})=\Psi_{2}(\mathrm{x}) \cdot \Gamma_{2}(\mathrm{t}) \tag{10}
\end{equation*}
$$

Where $\Psi_{1}(\mathrm{x}), \Psi_{2}(\mathrm{x}), \Gamma_{1}(\mathrm{t})$ and $\Gamma_{2}(\mathrm{t})$ are four unknown functions. By substituting equations (9) and (10) into equations (1) and (2), respectively, we obtain:

$$
\begin{align*}
& T_{1}=e^{-\beta_{n}^{2} t}\left(A_{1 n} \sin \frac{\beta_{n} x}{\sqrt{\alpha_{1}}}+B_{1 n} \cos \frac{\beta_{n} x}{\sqrt{\alpha_{1}}}\right)  \tag{11}\\
& T_{2}(x, t)=e^{-\beta_{n}^{2} t}\left(A_{2 n} \sin \frac{\beta_{n} x}{\sqrt{\alpha_{2}}}+B_{2 n} \cos \frac{\beta_{n} x}{\sqrt{\alpha_{2}}}\right) \tag{12}
\end{align*}
$$

While applying the boundary conditions, we have to solve the following system:

$$
\left\{\begin{array}{l}
0 \cdot B_{1 n}+A_{2 n} \cdot s_{4}+B_{2 n} \cdot s_{5}=0  \tag{13}\\
s_{1} \cdot B_{1 n}+A_{2 n} \cdot s_{2}-B_{2 n} \cdot s_{3}=0 \\
s_{6} \cdot B_{1 n}-A_{2 n} \cdot s_{3}-B_{2 n} \cdot s_{2}=0
\end{array}\right.
$$

with,

$$
\begin{aligned}
& \mathrm{K}=\frac{\lambda_{1} \sqrt{\alpha_{2}}}{\lambda_{2} \sqrt{\alpha_{1}}} ; \mathrm{s}_{1}=\mathrm{K} \cdot \sin \left(\frac{\mathrm{~B}_{\mathrm{n}} \mathrm{a}}{\sqrt{\alpha_{1}}}\right) ; \mathrm{s}_{2}=\cos \left(\frac{\mathrm{B}_{\mathrm{n}} \mathrm{a}}{\sqrt{\alpha_{2}}}\right) \\
& \mathrm{s}_{3}=\sin \left(\frac{\mathrm{B}_{\mathrm{n}} \mathrm{a}}{\sqrt{\alpha_{2}}}\right) ; \mathrm{s}_{4}=\mathrm{H} \cdot \sin \left(\frac{\mathrm{~B}_{\mathrm{n}}(\mathrm{a}+\mathrm{b})}{\sqrt{\alpha_{2}}}\right)-\cos \left(\frac{\mathrm{B}_{\mathrm{n}}(\mathrm{a}+\mathrm{b})}{\sqrt{\alpha_{2}}}\right) \\
& \mathrm{s}_{5}=\mathrm{H} \cdot \cos \left(\frac{\mathrm{~B}_{\mathrm{n}}(\mathrm{a}+\mathrm{b})}{\sqrt{\alpha_{2}}}\right)-\sin \left(\frac{\mathrm{B}_{\mathrm{n}}(\mathrm{a}+\mathrm{b})}{\sqrt{\alpha_{2}}}\right) ; \mathrm{s}_{6}=\cos \left(\frac{\mathrm{B}_{\mathrm{n}} \mathrm{a}}{\sqrt{\alpha_{1}}}\right) ; \mathrm{H}=\frac{\mathrm{h}_{2} \sqrt{\alpha_{2}}}{\lambda_{2} \beta_{\mathrm{n}}}
\end{aligned}
$$

We have now the following system:

$$
\left[\begin{array}{lll}
0 & s_{4} & s_{5}  \tag{14}\\
s_{1} & s_{2} & -s_{3} \\
s_{6} & -s_{3} & -s_{2}
\end{array}\right]\left[\begin{array}{c}
\mathrm{B}_{1 \mathrm{n}} \\
\mathrm{~A}_{2 \mathrm{n}} \\
\mathrm{~B}_{2 \mathrm{n}}
\end{array}\right]=\left[\begin{array}{l}
0 \\
0 \\
0
\end{array}\right]
$$

which contains four equations with five unknown coefficients: $A_{1 n}, B_{1 n}, A_{2 n}, B_{2 n}$ and $\beta_{n}$. The solution of this system (14) consists to take $B_{1 n}=1$, and to determine the others coefficients. Thus, the system of equations (13) to solve is now

$$
\left[\begin{array}{lll}
0 & s_{4} & s_{5}  \tag{15}\\
s_{1} & s_{2} & -s_{3} \\
s_{6} & -s_{3} & -s_{2}
\end{array}\right]\left[\begin{array}{c}
1 \\
A_{2 n} \\
B_{2 n}
\end{array}\right]=\left[\begin{array}{l}
0 \\
0 \\
0
\end{array}\right]
$$

To determine $A_{2 n}$ and $B_{2 n}$, we will only take two equations. We will have to solve the following system:

$$
\left[\begin{array}{ll}
s_{2} & -s_{3}  \tag{16}\\
-s_{3} & -s_{2}
\end{array}\right]\left[\begin{array}{l}
A_{2 n} \\
B_{2 n}
\end{array}\right]=\left[\begin{array}{l}
-s_{1} \\
-s_{6}
\end{array}\right] \quad \Leftrightarrow \quad\left[\begin{array}{ll}
-s_{2} & s_{3} \\
s_{3} & s_{2}
\end{array}\right]\left[\begin{array}{l}
A_{2 n} \\
B_{2 n}
\end{array}\right]=\left[\begin{array}{l}
s_{1} \\
s_{6}
\end{array}\right]
$$

We are going to use the determinant method:

$$
\begin{align*}
& A_{2 n}=s_{6} \cdot s_{3}-s_{1} \cdot s_{2}  \tag{17}\\
& B_{2 n}=s_{2} \cdot s_{6}+s_{1} \cdot s_{3}
\end{align*}
$$

In order to know the $\beta_{\mathrm{n}}$ arguments, we must cancel the determinant of the equation (16), which cannot be solved analytically. For that, we have used the numerical method, namely, bisection method, in order to calculate $\beta_{n}$ [9]. By knowing the $\beta_{n}$, we can easy deduce the coefficients $A_{i n}$ and $B_{i n}$, i.e., the temperatures distributions.

$$
\left[\begin{array}{lll}
0 & s_{4} & s_{5}  \tag{18}\\
s_{1} & s_{2} & -s_{3} \\
s_{6} & -s_{3} & -s_{2}
\end{array}\right]=0 \quad \Leftrightarrow \quad-s_{4} \cdot\left[\begin{array}{ll}
s_{1} & -s_{3} \\
s_{6} & -s_{2}
\end{array}\right]+s_{5} \cdot\left[\begin{array}{ll}
s_{1} & s_{2} \\
s_{6} & -s_{3}
\end{array}\right]=0
$$

What comes back to solve the next equation:

$$
\begin{equation*}
\mathrm{F}_{\mathrm{c}}=\mathrm{s}_{4} \cdot\left(\mathrm{~s}_{1} \cdot \mathrm{~s}_{2}-\mathrm{s}_{6} \cdot \mathrm{~s}_{3}\right)-\mathrm{s}_{5} \cdot\left(\mathrm{~s}_{1} \cdot \mathrm{~s}_{3}-\mathrm{s}_{2} \cdot \mathrm{~s}_{6}\right)=0 \tag{19}
\end{equation*}
$$

In our case we have to calculate infinities of solutions, then the initial value is of a primordial importance.

## 4. PREDICTION OF TEMPERATURES

The temperatures distributions in the first plate are given by the equation (1), which must satisfy the initial conditions (7) and (8).

$$
\begin{equation*}
\mathrm{T}(\mathrm{x}, \mathrm{t})=\mathrm{e}^{-\beta_{\mathrm{n}}^{2} \mathrm{t}} \mathrm{f}_{\mathrm{i}}(\mathrm{x}) \quad \text { and } \quad \mathrm{f}_{\mathrm{i}}(\mathrm{x})=\sum_{\mathrm{n}} \mathrm{C}_{\mathrm{n}} . \Psi_{\mathrm{in}}(\mathrm{x}) \tag{20}
\end{equation*}
$$

where

$$
\begin{array}{ll}
\Psi_{\text {in }}(\mathrm{x})=\mathrm{A}_{\mathrm{in}} \cdot \sin \left(\frac{\beta_{\mathrm{n}} \mathrm{x}}{\sqrt{\alpha_{\mathrm{i}}}}\right)+\mathrm{B}_{\mathrm{in}} \cdot \cos \left(\frac{\beta_{\mathrm{n}} \mathrm{x}}{\sqrt{\alpha_{\mathrm{i}}}}\right) \\
\begin{array}{l}
\mathrm{f}_{\mathrm{i}}(\mathrm{x})=\mathrm{T}_{0}^{\prime} \\
\mathrm{f}_{\mathrm{i}}(\mathrm{x})=\mathrm{T}_{0}
\end{array} \quad 0 \leq \mathrm{x} \leq \varepsilon  \tag{21}\\
\text { pour } \mathrm{t}=0
\end{array}
$$

By applying the operator $\frac{\lambda_{i}}{\alpha_{i}} \cdot \int_{x_{i}}^{x_{i+1}} \Psi_{i r} . d x$ to the both sides of equation (20), we find

$$
\begin{equation*}
\sum_{i=1}^{M} \frac{\lambda_{i}}{\alpha_{i}} \cdot \int_{x_{i}}^{x_{i+1}} \Psi_{i r}(x) \cdot f_{i}(x) \cdot d x=\sum_{n} C_{n} \cdot\left[\sum_{I+1}^{M} \frac{\lambda_{i}}{\alpha_{i}} \cdot \int_{x_{i}}^{x_{i+1}} \Psi_{i n} \cdot \Psi_{i n} \cdot d x\right] \tag{22}
\end{equation*}
$$

Where $M$ is the number of walls (in our case, $M=2$ ), and $x_{i}$ is the position of the wall in the selected reference.

The orthogonality and $\mathrm{N}_{\mathrm{n}}$ expressions are defined, respectively, as follows:

$$
\sum_{i=1}^{M} \frac{\lambda_{i}}{\alpha_{i}} \cdot \int_{x_{i}}^{x_{i+1}} \Psi_{i n} \cdot \Psi_{i r} \cdot d x= \begin{cases}0 & \text { if } n \neq r  \tag{23}\\ N_{n} & \text { if } n=r\end{cases}
$$

and

$$
\begin{equation*}
\mathrm{N}_{\mathrm{n}}=\sum_{\mathrm{j}=1}^{\mathrm{M}} \frac{\lambda_{\mathrm{i}}}{\alpha_{\mathrm{j}}} \cdot \int_{\mathrm{x}_{\mathrm{i}}}^{\mathrm{x}_{\mathrm{i}+1}} \Psi_{\mathrm{jn}}^{2} \mathrm{dx} \tag{24}
\end{equation*}
$$

By substituting equation (23) into equation (22), we obtain:

$$
\begin{equation*}
C_{n}=\frac{1}{\mathrm{~N}_{\mathrm{n}}} \cdot \sum_{\mathrm{i}=1}^{\mathrm{M}} \frac{\lambda_{\mathrm{i}}}{\alpha_{\mathrm{i}}} \cdot \int_{\mathrm{x}_{\mathrm{i}}}^{\mathrm{x}_{\mathrm{i}+1}} \Psi_{\mathrm{in}} \cdot \mathrm{f}_{\mathrm{i}}(\mathrm{x}) \mathrm{dx} \tag{25}
\end{equation*}
$$

Finally, the temperatures expressions is now

$$
\begin{equation*}
\mathrm{T}_{\mathrm{i}}(\mathrm{x}, \mathrm{t})=\sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{\mathrm{e}^{-\beta_{\mathrm{n}}^{2} \mathrm{t}}}{\mathrm{~N}_{\mathrm{n}}} \cdot \Psi_{\mathrm{in}} \cdot \sum_{\mathrm{i}=1}^{\mathrm{M}} \frac{\lambda_{\mathrm{i}}}{\alpha_{\mathrm{i}}} \cdot \int_{\mathrm{x}_{\mathrm{i}}}^{\mathrm{x}_{\mathrm{i}+1}} \Psi_{\mathrm{in}} \cdot \mathrm{f}_{\mathrm{i}}(\mathrm{x}) \cdot \mathrm{dx} \tag{26}
\end{equation*}
$$

We applies the initial conditions (7) and (8) to the above expression, we will have:

$$
\begin{gather*}
T_{1}(x, t)=\sum_{n=1}^{\infty} \frac{1}{N_{n}} \cdot e^{-\beta_{n}^{2} t} \cdot \cos \left(\frac{\beta_{n} x}{\sqrt{\alpha_{1}}}\right) \cdot G \quad \text { for } 0 \leq x \leq a  \tag{27}\\
T_{2}(x, t)=\sum_{n=1}^{\infty} \frac{1}{N_{n}} \cdot e^{-\beta_{n}^{2} t} \cdot\left(A_{2 n} \cdot \sin \left(\frac{\beta_{n} x}{\sqrt{\alpha_{2}}}\right)+B_{2 n} \cdot \cos \left(\frac{\beta_{n} x}{\sqrt{\alpha_{2}}}\right)\right) \cdot G \quad \text { for } \quad a<x \leq a+b \tag{28}
\end{gather*}
$$

By setting carries out the solution of the equation (26):

$$
\begin{array}{r}
N_{n}=w_{1}+w_{2}+w_{3}+w_{4} \text { and } G=G_{1}+G_{2}+G_{3}+G_{4} \\
G_{1}=\frac{\lambda_{1}}{\alpha_{1}} \cdot \int_{0}^{\varepsilon} T_{0} \cdot \cos \left(\frac{\beta_{\mathrm{n}} \mathrm{x}}{\sqrt{\alpha_{1}}}\right) \cdot d x=\frac{\lambda_{1} \cdot T_{0}}{\sqrt{\alpha_{1}} \cdot \beta_{\mathrm{n}}} \cdot \sin \left(\frac{\beta_{\mathrm{n}} \varepsilon}{\sqrt{\alpha_{1}}}\right) \\
\mathrm{G}_{2}=\frac{\lambda_{1}}{\alpha_{1}} \cdot \int_{\varepsilon}^{\mathrm{a}} \mathrm{~T}_{0}^{\prime} \cdot \cos \left(\frac{\beta_{\mathrm{n}} \mathrm{x}}{\sqrt{\alpha_{1}}}\right) \cdot \mathrm{dx}=\frac{\lambda_{1} \cdot T_{0}^{\prime}}{\sqrt{\alpha_{1}} \cdot \beta_{\mathrm{n}}} \cdot\left(\sin \left(\frac{\beta_{\mathrm{n}} \mathrm{a}}{\sqrt{\alpha_{1}}}\right)-\sin \left(\frac{\beta_{\mathrm{n}} \varepsilon}{\sqrt{\alpha_{1}}}\right)\right) \\
\mathrm{G}_{3}=\frac{\lambda_{2}}{\alpha_{2}} \cdot \int_{\mathrm{a}}^{\mathrm{a}+\mathrm{b}} \mathrm{~T}_{0}^{\prime} \cdot \mathrm{A}_{2 \mathrm{n}} \cdot \sin \left(\frac{\beta_{\mathrm{n}} \mathrm{x}}{\sqrt{\alpha_{2}}}\right) \cdot \mathrm{dx}=\frac{\lambda_{2} \cdot \mathrm{~T}_{0}^{\prime} \cdot \mathrm{A}_{2 \mathrm{n}}}{\sqrt{\alpha_{2}} \cdot \beta_{\mathrm{n}}} \cdot\left(\cos \left(\frac{\beta_{\mathrm{n}} \mathrm{a}}{\sqrt{\alpha_{2}}}\right)-\cos \left(\frac{\beta_{\mathrm{n}}(\mathrm{a}+\mathrm{b})}{\sqrt{\alpha_{2}}}\right)\right) \\
\mathrm{G}_{4}=\frac{\lambda_{2}}{\alpha_{2}} \cdot \int_{\mathrm{a}}^{\mathrm{a}+\mathrm{b}} \mathrm{~T}_{0}^{\prime} \cdot \mathrm{B}_{2 \mathrm{n}} \cdot \cos \left(\frac{\beta_{\mathrm{n}} \mathrm{x}}{\sqrt{\alpha_{2}}}\right) \cdot \mathrm{dx}=\frac{\lambda_{2} \cdot \mathrm{~T}_{0}^{\prime} \cdot B_{2 \mathrm{n}}}{\sqrt{\alpha_{2}} \cdot \beta_{\mathrm{n}}} \cdot\left(\sin \left(\frac{\beta_{\mathrm{n}}(\mathrm{a}+\mathrm{b})}{\sqrt{\alpha_{2}}}\right)-\sin \left(\frac{\beta_{\mathrm{n}} \mathrm{a}}{\sqrt{\alpha_{2}}}\right)\right) \tag{29}
\end{array}
$$

and

$$
\begin{aligned}
& \mathrm{w}_{1}=\frac{\lambda_{1}}{2 \alpha_{1}}\left(\mathrm{a}+\frac{\sqrt{\alpha_{1}}}{2 \beta_{\mathrm{n}}} \cdot \sin ^{2}\left(\frac{\beta_{\mathrm{n}} \mathrm{a}}{\sqrt{\alpha_{1}}}\right)\right) \\
& \mathrm{w}_{2}=\frac{\lambda_{2} \cdot \mathrm{~A}_{22_{\mathrm{n}}}}{2 \alpha_{2}}\left(\mathrm{~b}-\frac{\sqrt{\alpha_{2}}}{2 \beta_{\mathrm{n}}} \cdot\left(\sin ^{2}\left(\frac{\beta_{\mathrm{n}}(\mathrm{a}+\mathrm{b})}{\sqrt{\alpha_{2}}}\right)-\sin ^{2}\left(\frac{\beta_{\mathrm{n}} \mathrm{a}}{\sqrt{\alpha_{2}}}\right)\right)\right)
\end{aligned}
$$

$$
\begin{align*}
& \mathrm{w}_{3}=\frac{\lambda_{2} \cdot \mathrm{~B}_{2 \mathrm{n}}}{2 \alpha_{2}}\left(\mathrm{~b}+\frac{\sqrt{\alpha_{2}}}{2 \beta_{\mathrm{n}}} \cdot\left(\sin ^{2}\left(\frac{\beta_{\mathrm{n}}(\mathrm{a}+\mathrm{b})}{\sqrt{\alpha_{2}}}\right)-\sin ^{2}\left(\frac{\beta_{\mathrm{n}} \mathrm{a}}{\sqrt{\alpha_{2}}}\right)\right)\right) \\
& \mathrm{w}_{4}=\frac{\lambda_{2}}{\sqrt{\alpha_{2}}} \frac{\mathrm{~A}_{2 \mathrm{n}} \cdot \mathrm{~B}_{2 \mathrm{n}}}{\beta_{\mathrm{n}}}\left(\sin ^{2}\left(\frac{\beta_{\mathrm{n}}(\mathrm{a}+\mathrm{b})}{\sqrt{\alpha_{2}}}\right)-\sin ^{2}\left(\frac{\beta_{\mathrm{n}} \mathrm{a}}{\sqrt{\alpha_{2}}}\right)\right) \tag{30}
\end{align*}
$$

## 5. PRÉDICTION OF THE THERMAL CONTACT RÉSISTANCE

In this section, we can compute the thermal contact resistance by knowing the following temperatures: $\mathrm{T}_{1}(\varepsilon, \mathrm{t})=\mathrm{T}_{1}$ and $\mathrm{T}_{2}(\mathrm{a}+\mathrm{b}, \mathrm{t})=\mathrm{T}_{2}$, which are deduced from equations (27) and (28), respectively.

By knowing that:

$$
\begin{equation*}
\frac{\mathrm{R}_{\mathrm{t} 1}+\mathrm{R}_{\mathrm{t} 2}+\mathrm{R}_{\mathrm{c}}}{\mathrm{R}_{\mathrm{t} 1}+\mathrm{R}_{\mathrm{t} 2}}=\frac{\mathrm{T}_{1}-\mathrm{T}_{2 \mathrm{~m}}}{\mathrm{~T}_{1}-\mathrm{T}_{2}} \quad \text { and } \quad \mathrm{R}_{\mathrm{c}}=\left(\frac{\mathrm{T}_{1}-\mathrm{T}_{2 \mathrm{~m}}}{\mathrm{~T}_{1}-\mathrm{T}_{2}}-1\right) \cdot\left(\mathrm{R}_{\mathrm{t} 1}+\mathrm{R}_{\mathrm{t} 2}\right) \tag{31}
\end{equation*}
$$

with

$$
\begin{equation*}
\mathrm{R}_{\mathrm{t} 1}=\frac{\mathrm{a}-\varepsilon}{\lambda_{1}} \quad \text { and } \quad \mathrm{R}_{\mathrm{t} 2}=\frac{\mathrm{b}}{\lambda_{2}} \tag{32}
\end{equation*}
$$

Our computer code in Fortran language, which permits to compute the temperatures of both faces of composite plate, and the thermal contact resistance, if the extern temperature is measured. For more detail, see the flow chart illustrated in Fig. 2.


Fig. 2: Flow chart of the computational programme

## 6. RESULTS AND DISCUSSION

The temperatures distributions obtained from the analytic expressions are represented in the figures 3 and 4, for the both plates, respectively. Note that the first and second plates are composed by the aluminium and copper. We can see from these figures, that the temperature $\mathrm{T}_{1}(\varepsilon, \mathrm{t})$ decreases from $175^{\circ} \mathrm{C}$ at $\mathrm{t}=0.1 \mathrm{~s}$, to $25^{\circ} \mathrm{C}$ at $\mathrm{t}=10 \mathrm{mn}$. After, the steady state is reaches after long lime, which is about $20^{\circ} \mathrm{C}$ (Fig. 3). On the other hand, the temperature $\mathrm{T}_{2}(\mathrm{a}+\mathrm{b}, \mathrm{t})$ increases from $20^{\circ} \mathrm{C}$ until reaching a maximum value of $25^{\circ} \mathrm{C}$ at $\mathrm{t}=10 \mathrm{mn}$, then it decrease to reach the temperature limiting of $20^{\circ} \mathrm{C}$ at $\mathrm{t}=3.5 \mathrm{~h}$ (Fig. 4).


Fig. 3: Temperature of the first plate via the time, for two plates. The first and second plates are composed of aluminium and copper, respectively. Here, $a=0.1 \mathrm{~m}, \mathrm{~b}=0.05 \mathrm{~m}$


Fig. 4: Temperature of the second plate via the time, for two plates. The first and second plates are composed of aluminium and copper, respectively. Here, $a=0.1 \mathrm{~m}, \mathrm{~b}=0.05 \mathrm{~m}$.

Figure 5 shows the distribution of thermal resistances via the thermal conductivity $\lambda$ of the second plate. We can see from this figure, that the thermal resistances decrease, and have a very small values for high values of $\lambda$.


Fig. 5: Thermal resistances variation via the thermal conductivity of the second plate. Here, the initial temperature is maintained at $860^{\circ} \mathrm{C}$.

Figure 6 shows the distribution of thermal contact resistance via the heat transfer coefficient. We can sec also from this figure, that the thermal résistance decreases, and has a very small values for high values of heat transfer coefficient.


Fig. 6: Thermal resistances variation via the heat transfer coefficient. The first and second plate arc composed of aluminium and copper, respectively. Here, $a=0.05 \mathrm{~m}, \mathrm{~b}=0.05 \mathrm{~m}$. The initial temperature is maintained at $860^{\circ} \mathrm{C}$.

## 7. CONCLUSION

In this study, we have obtained analytically the temperatures expressions from the heat conduction equation for a plate with multi-layer by the method of separation-of-variables. The phenomenon of the thermal contact resistance remains one of the most complicated phenomena in
heat transfer problems, and so far, it is not modelled. Thus is why, we hope that our work will be a first step to develop the experimental process and thus to arrive, perhaps, to understand the parameters which influence this phenomenon.

## NOMENCLATURE

| $\mathrm{A}_{\mathrm{in}}$ | Constant |
| :--- | :--- |
| a | Thickness of the first plate (m) |
| cos | Cosinus |
| $\mathrm{C}_{\mathrm{p}}$ | Specific heat |
| $\mathrm{C}_{\mathrm{n}}$ | Constant |
| h | Convective heat transfer coefficient |
| N | Norm |
| Q | Heat quantity $(\mathrm{W})$ |
| s in | Sinus |
| T | Temperature $\left({ }^{\circ} \mathrm{C}\right)$ |
| t | Time $(\mathrm{s})$ |
| $\alpha$ | Thermal diffusivity $\left(\mathrm{m}^{2} / \mathrm{s}\right)$ |
| $\lambda$ | Thermal conductivity $(\mathrm{w}$ |
| $\rho$ | Density $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ |
| $\Psi$ | Space function |
| c | Contact |
| M | Numbers of walls |
| m | Measured |

$\mathrm{B}_{\text {in }}$ Constant
b Thickness of the second plate (m)
e Exponential function
G Function
H Constant
K Constant
n Integer number
R Thermal contact résistance
w Constant
x Coordinate (m)

## Symbols greeks

Constant of integration
Low thick (m)
Time function* Indices
First wall
Second wall

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