Rev. Energ. Ren.Vol.7 (2004) 53 -71

Infinite Reflections Method for Calculation of Radiation Exchange between Grey-Diffuse Surfaces : *UP to four surface interactions* (Part 1)

A. Bouchair

Département d'Architecture, Faculté des sciences de l'ingénieur, Université de Jijel, Algérie
E-mail: abouchair@mail.univ-jijel.dz ou abouchair@yahoo.fr

Abstract –- *Unlike black surfaces, calculation of radiation exchange for grey-diffuse surfaces is generally considered too complex. This is because a grey surface is not a perfect absorber as for a black surface. As radiation leaves a surface, it travels to the other surfaces whereby it is absorbed partially and is then reflected many times in between with partial absorption at each contact with a surface. Therefore, a proper analysis of the problem must take into account of these multiple reflections. The existing methods of analysis can be classified into two categories; those which do not take into account of the multiple reflections for more than two surfaces, and those which account for these multiple reflections but they are only applicable for enclosures and are not fully reliable in implementation. This first part of the paper presents a method of analysis for the exact calculation of radiant heat transfer between two grey-diffuse surfaces due to the interaction of up to four surfaces whereby multiple reflections of radiation are accounted for. This is based upon two concepts; the arborescent diagrams and the infinite series algebra. Complementary work for the understanding of this part is presented in the second part of the paper.*

Résumé – *Contrairement aux surfaces noires, le calcul d'échange radiatif pour des surfaces grises et diffuses est considéré généralement trop complexe. C'est parce qu'une surface grise n'est pas un absorbeur parfait comme pour le cas d'une surface noire. Quand une radiation quitte une surface, elle voyage aux autres surfaces par lesquelles elle est absorbée partialement et est renvoyée plusieurs fois entre ces surfaces avec absorption partielle à chaque contact. Par conséquent, une analyse adéquate du problème doit prendre en considération de ces réflexions multiples. Les méthodes existantes d'analyse peuvent être classées dans deux catégories; ce qui ne prennent pas en considération des réflexions multiples pour plus de deux surfaces, et ce qui compte pour ces réflexions multiples mais ils sont seulement applicables pour les volumes fermés et ne sont pas complètement fiable dans ses mise en oeuvre. Cet article présente une méthode d'analyse pour le calcul exact de transfert de la chaleur radiante entre deux surfaces grises et diffuses dû à l'interaction de jusqu'à quatre surfaces par lequel les réflexions multiples de radiation sont tenues en comptes. Cela est basé sur deux concepts; les diagrammes arborescents et l'algèbre des séries infinies. Pour la compréhension de cette partie, un travail complémentaire est présenté dans la deuxième partie de l'article.*

Key words: Multiple reflections of radiation – Radiant heat transfer – Grey-diffuse surfaces – Low emissivity – Arborescent diagrams – infinite series – Four planar surfaces interaction.

1. INTRODUCTION

Calculation of radiant heat exchange for grey surfaces, especially when emissivities are low, is a complicated problem. As radiation leaves a surface, it travels to the other surfaces whereby it is absorbed partially and is then reflected many times within the enclosure with partial absorption at each contact with a surface. If the enclosed surfaces are open to external environment, part of these radiations will be reflected out of the system entirely. The process of reflection and absorption continue until all radiation is fully absorbed by surfaces. Neglecting these multiple reflections will cause significant errors in the calculation of radiant heat exchange.

Several methods of analysis for radiant heat exchange between surfaces of an enclosure have been found in the literature [1, 2, 3, 4]. The most important ones are; the configuration factor method introduced by Hottel [1, 2] and the network method introduced by Oppenheim [3]. Both methods are basically equivalent for simple problems which do not involve many surfaces. However, The network method of Oppenheim is more developed and convenient for problems which involve many surfaces. For more detailed information about the network method see references [5, 6, 7].

In building energy applications, the estimation of radiation heat transfer between two grey surfaces, of areas A_1 and A_2, emissivities ε_1 and ε_2, maintained at absolute temperatures T_1 and T_2, is generally made by the following equation:

$$Q_{(2)1 \leftrightarrow 2} = \sigma . A_1 . \left(T_1^4 - T_2^4 \right) . C_{12} \tag{1}$$

This can be easily linearised, to be used in conjunction with other mode of heat transfer such as convection and conduction, into the following form:

$$Q_{(2)1 \leftrightarrow 2} = h_{r12} . A_1 . \left(T_1 - T_2 \right) \tag{2}$$

$C_{12}$is a dimensionless *effective configuration factor for grey surfaces*, first introduced by Hottel [1] which depends upon the emissivity of each surface and the geometrical configuration of the surfaces specified by the *configuration factor for black surfaces* F_{12}.

$$C_{12} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1}{F_{12}} + \frac{A_1}{A_2} . \left(\frac{1-\varepsilon_2}{\varepsilon_2} \right)} \tag{3}$$

For the calculation of F_{12} see references [2,5,8 and 9]. The weakness of this equation is that it neglects the effect of multiple reflections of radiation between surfaces other than surface 1 and 2 which in fact will led to significant errors in the calculations. Crabol [4] has also derived an equation where he accounted for multiple reflections but for radiation exchange between two finite planar grey-surfaces.

$$Q_{(2)1 \leftrightarrow 2} = \frac{\varepsilon_1 . \varepsilon_2 . \sigma . A_1 . \left(T_1^4 - T_2^4 \right) . F_{12}}{1 - \left(1 - \varepsilon_1 \right) \left(1 - \varepsilon_2 \right) F_{12} . F_{21}} \tag{4}$$

To overcome the problem of the effect of all surfaces of an enclosure, the network method or "the radiosity matrix method" (N.W.M) treats the problem of radiation exchange differently for which it introduces the concepts of radiosity, surface and space resistance to radiation. These are the basis to the construction of an equivalent network to represent the interaction of surfaces in question. The radiosity is the sum of the energy emitted and the energy reflected when no energy is transmitted. For more details of the method see also references [4, 6, 7 and 10].

For an enclosure consisting of several surfaces or "zones" with prescribed temperatures T_i for each surface (i = 1, 2, …. N), of areas A_i and emissivities ε_i , the radiation heat transfer from any one of them can be calculated by the solution of an algebraic matrix equation for the unknown radiosities J_i which can be formulated from the following expression:

$$\frac{1}{\varepsilon_i} J_i - \frac{1-\varepsilon_i}{\varepsilon_i} \sum_{j=1}^{N} J_j . F_{ij} = \sigma . T_i^4 \tag{5}$$

Equation (5) can be written for each of the N surfaces of the enclosure giving N equations for N unknowns. This can be, for convenience, expressed in matrix form as:

$$\left[M_{ij} \right] \cdot \left[J_i \right] = \left\lfloor \sigma . T_i^4 \right\rfloor \tag{6}$$

Where $\left[J_i \right]$ is the radiosity vector, $\left\lfloor \sigma . T_i^4 \right\rfloor$ is the surface input vector and $\left[M_{ij} \right]$ is the N x N coefficient matrix;

$$M_{ij} = \frac{a_{ij} - (1 - \varepsilon_i) F_{ij}}{\varepsilon_i} \tag{7}$$

$a_{ij} = 1$ for $i = j$, $a_{ij} = 0$ for $i \neq j$

Once the radiosity of each surface is obtained from the solution of the resulting matrix q_i, which is in fact, the net rate of heat loss or gain per unit area at surface, i, due to all surface interactions is calculated from the following equation:

$$q_i = Q_i \big/ A_i = \frac{\varepsilon_i}{1 - \varepsilon_i} \left(\sigma . T_i^4 - J_i \right) \tag{8}$$

Equations (5) and (8) from the network method are restricted only for surfaces with emissivities $\varepsilon_i \neq 0$ and $\varepsilon_i \neq 1$. That is the reason why, the network method change the approach of formulation of the problem when surface emissivities $\varepsilon_i = 0$ and $\varepsilon_i = 1$ are involved. For a re-radiating surface, that is when ($\varepsilon_i = 1$), the net heat flux q_i on that surface is zero and thus $\sigma . T_i^4 = J_i$. This approach considers an enclosure where temperatures $T_i$are prescribed for some of the surfaces ($i = 1, 2, …., k$), and the net heat fluxes q_i for the remaining surfaces ($i = k+1, k+2, …., N$) the N equations for determination of N unknown radiosities J_i ($i = 1, 2, …., N$) are obtained as follows:

For surfaces $i = 1, 2, …., k$ with prescribed surface temperatures we use equation (5).

For surfaces or "zones" $i = k+1, k+2, …., N$ with prescribed net heat fluxes, we use the following equation:

$$J_i - \sum_{j=1}^{N} J_j . F_{ij} = q_i \tag{9}$$

Equation (5) and (9) can for computational purposes be written in matrix form as shown in (6). Where for $i = 1, 2, …., k$, M_{ij} is given by equation (7). For $i = k+1, k+2, …., N$:

$$M_{ij} = a_{ij} - F_{ij} \tag{10}$$

For i=j, $a_{ij} = 1$ and for $i \neq j$, $a_{ij} = 0$

Although the network method does solve the problem of the effect of several surfaces in the calculation of radiation heat transfer, it has obvious limitatinons in that;

- it is valid only for surfaces forming an enclosure where ($F_{i2} + F_{i3} + …. = 1$),
- it requires computer implementation and matrix knowledge for the solution of the system of equations,

- the method, however, calculates the net radiation loss or gain at each surface resulted from the interaction of the surrounding surfaces and will not directly give net radiation exchange between each two surfaces as does equation (1) from which an effective radiation coefficient can be calculated. In other words, the network method will not allow linearisation of radiative heat transfer to be used with other modes of heat transfer.

The present paper attempts to establish an analytical method for the calculation of radiant heat transfer between two grey-diffuse surfaces due to the interaction of up to four surfaces. The analysis is called Infinite Reflection Method or "I.R.M". The strategy of the method is based upon two fundamental concepts; the arborescent diagrams and the infinite series algebra which greatly facilitate the analysis. In order to reduce the length of the paper, it was divided into two parts. The first part treats theoretical foundations of the method. The second part presents the diagrams and the derivation of coefficients and factors.

2. ANALYSIS OF RADIATION BY INFINITE REFLECTIONS METHOD

In order to simplify the analysis, it is reasonably acceptable to assume that:

All surfaces are grey : the emissivity, absorptivity and reflectivity are independent of wavelength but they depend on surface temperature. Under the grey body assumption, that is, $\varepsilon_\lambda = \varepsilon$ = constant, the absoptivity and emissivity can be related by Kirchhoff law as: $\varepsilon = \alpha$

All surfaces are diffuse in emission and reflection, for which the emissivity and the absorptivity are independent of the direction, that is, ($\varepsilon_\theta = \varepsilon$ and $\varepsilon_{\theta,\lambda} = \varepsilon_\lambda$) and ($\alpha_\theta = \alpha$ and $\alpha_{\theta,\lambda} = \alpha_\lambda$). The intensity of radiation leaving a diffuse surface is uniform in all directions.

So that, geometric configuration factors derived for black surfaces can also be used for grey-diffuse surfaces [9].

The temperature is uniform over each surface (each surface is assumed to be at its mean temperature)

The incident and the reflected energy flux is uniform over each surface

No interference of external radiation

To illustrate the method of approach for calculating radiation heat transfer between grey-diffuse surfaces, we first derive an expression for the rate of radiation heat transfer between two surfaces in a three surface enclosure.

2.1. Radiation exchange between two planar surfaces due to three-surface interaction:

Consider three opaque grey-diffuse planar surfaces 1, 2 and 3 that the end effects are negligible (figure 1) . Surface 1 of area A_1 and emissivity ε_1, is maintained at absolute temperature T_1. Surface 2 of area A_2 and emissivity ε_2, is maintained at absolute temperature T_2. Surface 3 has an emissivity ε_3. So that the radiant heat exchange between surface 1 and 2 with the interaction of a third surface can be exactly calculated. Figure 1 is a schematic diagram which shows the general mecanism and type of radiation exchange between the three surfaces, used in the analysis.

Surface 1 emit radiation $Q_{e,12}$ to surface 1 , $Q_{e,13}$ to surface 2 and reflects $Q_{r,12}$ to surface 2 and $Q_{r,13}$ to surface 3. It also absorbes a portion $Q_{a,1}$. A portion of emitted and reflected radiation $Q_{1,os}$ can get out of the system entirely if the surfaces do not form a perfect enclosure.

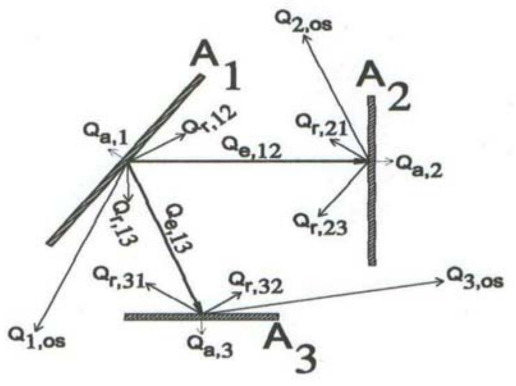


Fig. 1: Sketch for radiation exchange between two surfaces due to three-surface interaction

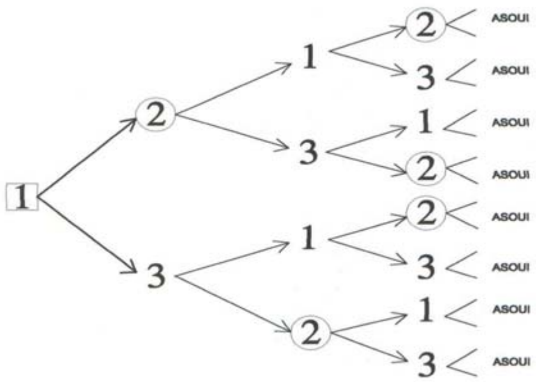
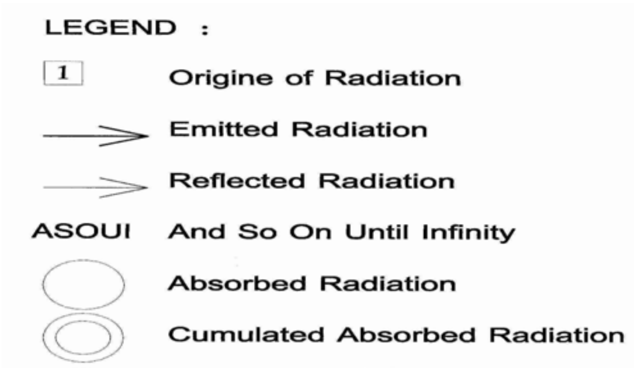


Fig. 2: Complete arborescent diagram for radiation exchanges between three surfaces.

Surface 2 reflects $Q_{r,21}$ to surface 1 and $Q_{r,23}$ to surface 3 and absorbes a portion $Q_{a,2}$. A portion of reflected radiation $Q_{2,OS}$ can get out of the system entirely if the surfaces do not form a perfect enclosure. Surface 3 reflects $Q_{r,31}$ to surface 1 and $Q_{r,32}$ to surface 2 and absorbs a portion $Q_{a,3}$. A portion of reflected radiation $Q_{3,OS}$ can get out of the system entirely if the surfaces do not form a perfect enclosure. If we follow the beams of radiation as they undergo the process of inter-reflection and absorption we will see that the radiant energy emitted by surface 1 that arrives at surface 2 will be reflected back and forth between the three surfaces several times with partial absorption at each contact with a surface.

Figure 2 is a complete arborescent diagram which shows the path that is followed by the radiation as it leaves surface 1. Each surface is represented by a numbered nod. Each nod in the diagram is divided into three branches (or arrows). The first two arrows in the diagram correspond to the emitted radiation from surface 1. The rest of arrows correspond to reflected radiation (see legend).



The radiant flux emitted by surface 1 is given by:

$$Q_1 = \varepsilon_1 . \sigma . A_1 . T_1^4 \tag{11}$$

With reference to figure 3, the fraction of radiation leaving surface 1 which arrives at surface 2 before it is reflected back from surface 2 is :

$$Q_{12} = Q_1 \left(F_{12} + X_1 + X_2 \right) \tag{12}$$

If we write equation (12) as:

$$Q_{12} = Q_1 . f_1 \tag{13}$$

therefore:

$$f_1 = F_{12} + X_1 + X_2 \tag{14}$$

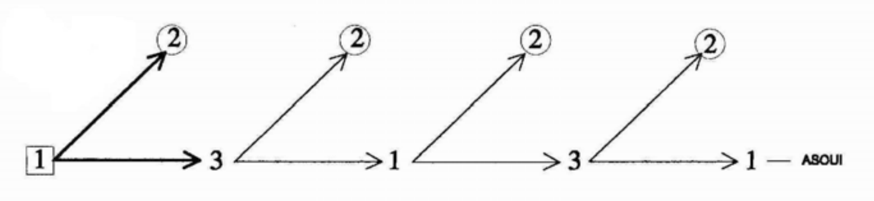


Fig. 3: Basic arborescent diagram for the fraction of radiation emitted by surface 1 that arrives at surface 2, used to derive f_1 factor for a three surface enclosure.

where,

$Q_1.F_{12}$ is the fraction of radiation leaving surface 1 which arrives at surface 2 directly whose path is (1→2).

$Q_1.X_1$ is the total radiation that leaves surface 1 to surface 3 and undergo the process of inter-reflection between them before it arrives at surface 2 at each time following the paths (1→3→2, 1→3→1→3→2, 1→3→1→3→1→3→2, and so on until infinity). Hence;

$$Q_1.X_1 = Q_{132} + Q_{13132} + Q_{131313132} + Q_{1313131313132} + \ldots.. \tag{15}$$

$$X_1 = F_{13}.F_{32}(1-\varepsilon_3) + F_{13}^2.F_{31}.F_{32}.(1-\varepsilon_1)(1-\varepsilon_3)^2 + F_{13}^3.F_{31}^2.F_{32}.(1-\varepsilon_1)^2(1-\varepsilon_3)^3 + \ldots\ldots\infty \tag{16}$$

Equation (16) is a geometric series with common ratio $CR_1 = (1-\varepsilon_1)(1-\varepsilon_3)F_{13}.F_{31}$ whose sum to infinity is:

$$X_1 = \frac{(1-\varepsilon_3)F_{13}.F_{32}}{1-CR_1} \tag{17}$$

$Q_1.X_2$ is the total radiation that leaves surface 1 and follow the following paths (1→3→1→2, 1→3→1→3→1→2, …and so on until infinity). Hence;

$$Q_1.X_2 = Q_{1312} + Q_{131312} + Q_{13131312} + \ldots.. \tag{18}$$

$$X_2 = F_{13}.F_{31}.F_{12}(1-\varepsilon_1)(1-\varepsilon_3) + F_{13}^2.F_{31}^2F_{12}(1-\varepsilon_1)^2(1-\varepsilon_3)^2 + F_{13}^3.F_{31}^3F_{12}(1-\varepsilon_1)^3(1-\varepsilon_3)^3 + \ldots\ldots\infty \tag{19}$$

Equation (19) is a geometric series (or geometric progression) with common ratio equals to CR_1, whose sum to infinity is:

$$X_2 = \frac{(1-\varepsilon_1)(1-\varepsilon_3)F_{13}.F_{31}.F_{12}}{1-CR_1} \tag{20}$$

where $|CR_1| < 0$

Substituting x_1 and x_2 by their values from equations (17) and (20) we can write:

$$X_1 + X_2 = \frac{F_{13}.(1-\varepsilon_3)[F_{32} + (1-\varepsilon_1)F_{31}.F_{12}]}{1-CR_1} \tag{21}$$

With reference to figure 4, the fraction of radiation reflected from surface 2 back to surface 1 is:

$$Q_{21} = Q_{12}\left(F_{21} + Y_1 + Y_2\right)\left(1-\varepsilon_2\right) \tag{22}$$

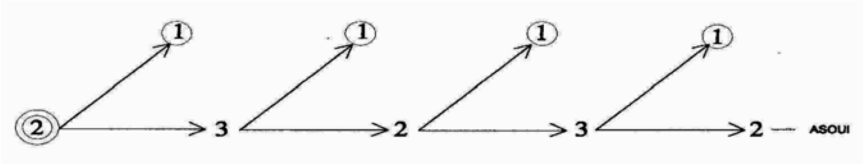


Fig. 4: Basic arborescent diagram for the fraction of radiation reflected from surface 2 that arrives at surface 1, used to derive f_2 factor for a three surface enclosure.

Equation (22) can be written as follows:

$$Q_{21} = Q_{12}.f_2.\left(1-\varepsilon_2\right) \tag{23}$$

From where:

$$f_2 = F_{21} + Y_1 + Y_2 \tag{24}$$

where, $Q_{12}.F_{21}.\left(1-\varepsilon_2\right)$ is the fraction of radiation that is reflected from surface 2 and arrives at surface 1 directly whose path is (2→1). $Q_{12}.Y_1$ is the total radiation that leaves surface 2 to surface 3 and undergo the process of inter-reflection between them before it arrives at surface 1 at each time following the paths (2→3→1, 2→3→2→3→1, 2→3→2→3→2→3→1, …→∞). Hence;

$$Q_{12}.Y_1 = Q_{231} + Q_{23231} + Q_{232323231} + Q_{2323232323231} + \ldots\ldots \tag{25}$$

$$Y_1 = F_{23}.F_{31}\left(1-\varepsilon_3\right) + F_{23}^2.F_{32}.F_{31}.\left(1-\varepsilon_2\right)\left(1-\varepsilon_3\right)^2 + F_{23}^3.F_{32}^2.F_{31}.\left(1-\varepsilon_2\right)^2\left(1-\varepsilon_3\right)^3 + \ldots\ldots\infty \tag{26}$$

Equation (26) is a geometric series with common ratio $CR_2 = \left(1-\varepsilon_2\right)\left(1-\varepsilon_3\right)F_{23}.F_{32}$ whose sum to infinity is:

$$Y_1 = \frac{\left(1-\varepsilon_3\right)F_{23}.F_{31}}{1-CR_2} \tag{27}$$

$Q_{12}.Y_2$ is the total radiation that is reflected by surface 2 to surface 3 and returns back to surface 2 before it arrives at surface 1 at each time (2→3→2→1, 2→3→2→3→2→1, and so on until infinity). Hence;

$$Q_{12}.Y_2 = Q_{2321} + Q_{232321} + Q_{23232321} + \ldots\ldots \tag{28}$$

$$Y_2 = F_{23}.F_{32}.F_{21}\left(1-\varepsilon_2\right)\left(1-\varepsilon_3\right) + F_{23}^2.F_{32}^2.F_{21}\left(1-\varepsilon_2\right)^2\left(1-\varepsilon_3\right)^2 + \ldots\infty \tag{29}$$

Equation (29) is a geometric series with common ratio is equal to CR_2 whose sum to infinity is:

$$Y_2 = \frac{\left(1-\varepsilon_2\right)\left(1-\varepsilon_3\right)F_{23}.F_{32}.F_{21}}{1-CR_2} \tag{30}$$

where $\left|CR_2\right| < 0$. Substituting Y_1 and Y_2 by their values from equations (27) and (30), then we can write:

$$Y_1 + Y_2 = \frac{F_{23}.\left(1-\varepsilon_3\right)\left[F_{31} + \left(1-\varepsilon_2\right)F_{32}.F_{21}\right]}{1-CR_2} \tag{31}$$

The fraction of radiation first reflected from 1 back to 2 is:

$$Q_{1212} = Q_{21}.f_1.(1-\varepsilon_1) \tag{32}$$

using equation (23) and (13) into (32) yields:

$$Q_{1212} = Q_1.f_1^2.f_2(1-\varepsilon_1)(1-\varepsilon_2) \tag{33}$$

The radiant flux intercepted due to infinite reflections between 1 and 2, ($1\rightarrow2\rightarrow1\rightarrow2 \ldots\rightarrow\infty$). is:

$$Q_{1212\ldots\infty} = Q_1.\left[f_1^2.f_2(1-\varepsilon_1)(1-\varepsilon_2) + f_1^3.f_2^2(1-\varepsilon_1)^2(1-\varepsilon_2)^2 + \ldots\infty\right] \tag{34}$$

And therefore $Q_{12} + Q_{1212\ldots\infty}$ is a progression of common ratio $CR_3 = (1-\varepsilon_1)(1-\varepsilon_2)f_1.f_2$, thus:

$$Q_{12} + Q_{1212\ldots\infty} = \frac{Q_1.f_1}{1-CR_3} \tag{35}$$

with reference to figure 5 the total radiation that follows the path ($1\rightarrow2\rightarrow3\rightarrow2\rightarrow3\rightarrow2 \ldots\rightarrow\infty$):

$$\begin{aligned}\sum Q_{12\ldots3232\ldots\infty} &= Q_{123232\ldots\infty} + Q_{12123232\ldots\infty} + Q_{1212123232\ldots\infty} + \ldots\ldots\infty \\ &= (Q_{12} + Q_{1212\ldots\infty}).\left[F_{23}.F_{32}(1-\varepsilon_3)(1-\varepsilon_2) + F_{23}^2.F_{32}^2(1-\varepsilon_3)^2(1-\varepsilon_2)^2 + \ldots\infty\right]\end{aligned} \tag{36}$$

which can be written as:

$$\sum Q_{12\ldots3232\ldots\infty} = \left[\frac{Q_1.f_1}{1-CR_3}\right].\left[\frac{(1-\varepsilon_2)(1-\varepsilon_3)F_{23}.F_{32}}{1-CR_2}\right] = \frac{Q_1.f_1}{1-CR_3}.f_3 \tag{37}$$

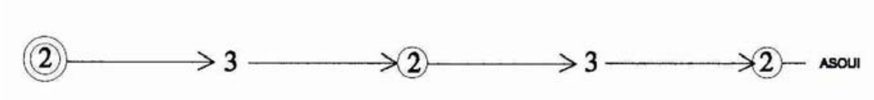


Fig. 5: Basic arborescent diagram for the fraction of radiation reflected from surface that arrives at another surface and back to itself, used to derive f_3 factor for a three surface enclosure.

The total radiation absorbed by surface A2 due to emission of surface A1 with the presence of surface A3 is:

$$Q^a_{(3)1\rightarrow2} = \varepsilon_2\left[Q_{12} + Q_{1212\ldots\infty} + \sum Q_{12\ldots3232\ldots\infty}\right] = \varepsilon_2.\left[\frac{Q_1.f_1}{1-CR_3}\right].\left[1+f_3\right] \tag{38}$$

If we let $f_3 = \dfrac{(1-\varepsilon_2)(1-\varepsilon_3)F_{23}.F_{32}}{1-(1-\varepsilon_2)(1-\varepsilon_3)F_{23}.F_{32}}$ and substitute Q_1 by its value from equation (10) then equation (38) becomes:

$$Q^a_{(3)1\rightarrow2} = \varepsilon_1.\varepsilon_2.\sigma.A_1.T_1^4.\left[\frac{f_1.(1+f_3)}{1-(1-\varepsilon_1)(1-\varepsilon_2)f_1.f_2}\right] \tag{39}$$

The net radiation exchange between the two surfaces can be determined as follows:
If we let

$$B = \frac{f_1.(1+f_3)}{1-(1-\varepsilon_1)(1-\varepsilon_2)f_1.f_2} \tag{40}$$

then, the total flux absorbed by surface 2 due to emission at A1 after the inclusion of all reflections can be written as:

$$Q^{a}_{(3)1\rightarrow 2} = \varepsilon_1.\varepsilon_2.\sigma.A_1.T_1^4.B \tag{41}$$

By interchanging subscripts 1 and 2, the flux absorbed by surface A1 due to emission at A2 is:

$$Q^{a}_{(3)2\rightarrow 1} = \varepsilon_1.\varepsilon_2.\sigma.A_2.T_2^4.C \tag{42}$$

where $C=\dfrac{f_2.\left(1+f_3^{'}\right)}{1-\left(1-\varepsilon_1\right)\left(1-\varepsilon_2\right)f_1.f_2}$ and $f_3^{'}=\dfrac{\left(1-\varepsilon_1\right)\left(1-\varepsilon_3\right)F_{13}.F_{31}}{1-\left(1-\varepsilon_1\right)\left(1-\varepsilon_3\right)F_{13}.F_{31}}$

The net flux exchange between A1 and A2 is:

$$Q_{(3)1\leftrightarrow 2} = Q^{a}_{(3)1\rightarrow 2} - Q^{a}_{(3)2\rightarrow 1} \tag{43}$$

but since $Q_{(3)1\leftrightarrow 2}$ must be zero when $T_1 = T_2$, therefore:

$$A_1.B = A_2.C \tag{44}$$

Hence for three surfaces interaction:

$$Q_{(3)1\leftrightarrow 2} = \varepsilon_1.\varepsilon_2.\sigma.A_1.\left(T_1^4 - T_2^4\right).B \tag{45}$$

In general for a number of surfaces (n ≥ 2), and by substituting B by its value from equation (40) we can write:

$$Q_{(n)1\leftrightarrow 2} = \varepsilon_1.\varepsilon_2.\sigma.A_1.\left(T_1^4 - T_2^4\right).\left[\frac{f_1.\left(1+f_3\right)}{1-\left(1-\varepsilon_1\right)\left(1-\varepsilon_2\right)f_1.f_2}\right] \tag{46}$$

For three surface enclosure n=3. The factors f_1, f_2 and f_3 depend on the number of interacting surfaces. The advantage of this equation is that the factors are only calculated once and allow for the linearization of the equation for direct use. In building energy applications, radiation heat transfer is often associated with other modes of heat transfer such as convection and conduction which are expressed in terms of simple linearised form. Accordingly, radiation heat transfer has to be linearised. The linearization requires to operate in terms of radiation coefficient h_r. This usually made by multiplying a linearised radiative heat transfer coefficient h_{r12} by the temperature difference between surfaces $\left(T_1 - T_2\right)$. Hence;

$$Q_{(n)1\leftrightarrow 2} = h_{r12}.A_1.\left(T_1 - T_2\right) = \varepsilon_1.\varepsilon_2.\sigma.A_1.\left(T_1^4 - T_2^4\right).B \tag{47}$$

$$h_{r12} = \frac{\varepsilon_1.\varepsilon_2.\sigma.\left(T_1^4 - T_2^4\right).B}{\left(T_1 - T_2\right)} = \varepsilon_1.\varepsilon_2.\sigma.\left(T_1^2 + T_2^2\right).\left(T_1^2 + T_2^2\right).B \tag{48}$$

and since $\left|Q_{(n)1\leftrightarrow 2}\right| = \left|Q_{(n)2\leftrightarrow 1}\right|$ therefore; $h_{r21} = \dfrac{A_1}{A_2} h_{r21}$

2.2. Radiation exchange between two planar surfaces due to two-surface interaction:

By neglecting the reflection of radiation from the third surface that is $1-\varepsilon_3 = 0$ which led to $f_1 = F_{12}$, $f_2 = F_{21}$ and $f_3 = 0$ so that equation (46) simplifies to Crabol equation (4).

2.3 Radiation exchange between two planar surfaces due to four-surface interaction:

The method of analysis outlined in section 2.1 can be extended to establish relationships to calculate radiation heat exchange between two surfaces in an enclosure containing four interacting surfaces. Equation (46) can be used for the calculation of radiant heat exchange between two surfaces of an enclosure consisting of four surfaces. However, the factors f_1, f_2 and f_3 has to be derived for this case. Figure 6 is a schematic diagram which shows the general mechanism and type of radiation exchange between the four surfaces, used in the analysis. Figure 7 is the complete arborescent diagram for radiation interchange corresponding to figure 6.

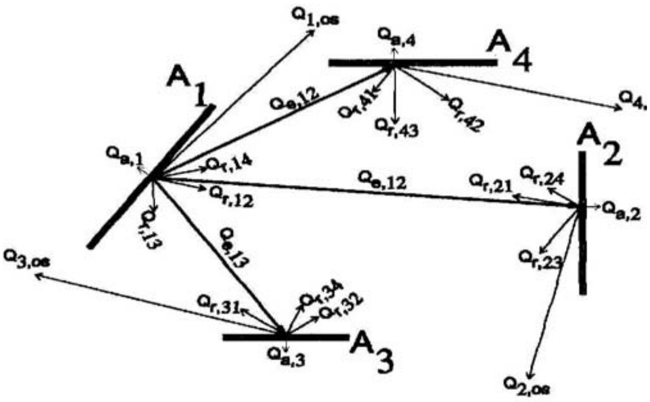


Fig. 6: Sketch for radiation exchange between two surfaces due to four-surfaces interaction

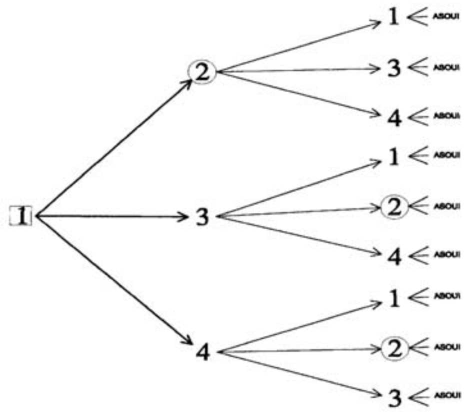


Fig. 7: Complete arborescent diagram for radiation exchanges between four surfaces.

Diagrams 1, 2 and 3 in Appendix 1 in the second part of this paper are basic arborescent diagrams used to derive the factors f_1, f_2 and f_3. A more detailed and developed diagram corresponding to diagram 1 is presented by diagram 4. Developed diagrams can similarly be established for other cases.

With reference to diagram 1in appendix 1 and by making use of equations (c56) and (c57) from appendix 4, we can write:

$$f_1 = F_{12} + \sum_{j=3}^{4} \frac{1}{1-CR}\left(X_1 + X_2 + X_3 + X_4 + \frac{R_0 + R'_0 + T_0 + T'_0}{1-CR_6}\right) \tag{49}$$

Where, for $j = 3$, $k = 4$, and for $j = 4$, $k = 3$ and for the derivation of the values of X_1, X_2, X_3, X_4, R_0, R'_0, T_0, T'_0, CR and CR_6 see appendix 2 and 4. Hence;

$$R_0 = \frac{(1-\varepsilon_1)(1-\varepsilon_j)^2.(1-\varepsilon_k)F_{1j}.F_{jk}.F_{k1}\left[F_{j2} + (1-\varepsilon_1)F_{j1}.F_{12}\right].\left[F_{1j} + (1-\varepsilon_j)F_{1k}.F_{kj}\right]}{[1-CR_4][1-CR_5]} \tag{50}$$

$$R'_0 = \frac{(1-\varepsilon_1)(1-\varepsilon_j)^2.(1-\varepsilon_k)^2.F_{1j}.F_{jk}.F_{k1}.V_1}{[1-CR_4][1-CR_5]^2} \tag{51}$$

where $V_1 = \left[F_{k2} + (1-\varepsilon_1)F_{k1}.F_{12}\right]\left[F_{jk} + (1-\varepsilon_1)F_{j1}.F_{1k}\right]\left[F_{1j} + \left(1-\varepsilon_j\right)F_{1k}.F_{kj}\right]$

$$T_0 = \frac{(1-\varepsilon_1)\left(1-\varepsilon_j\right)(1-\varepsilon_k)F_{1j}.F_{j1}\left[F_{k2} + (1-\varepsilon_1)F_{k1}.F_{12}\right]\left[F_{1k} + \left(1-\varepsilon_j\right)F_{1j}.F_{jk}\right]}{\left[1-CR_4\right]\left[1-CR_5\right]} \quad (52)$$

$$T_0' = \frac{(1-\varepsilon_1)\left(1-\varepsilon_j\right)^2 .(1-\varepsilon_k)F_{1j}.F_{j1}.V_2}{\left[1-CR_4\right]^2.\left[1-CR_5\right]} \quad (53)$$

where $V_2 = \left[F_{j2} + (1-\varepsilon_1)F_{j1}.F_{12}\right]\left[F_{kj} + (1-\varepsilon_1)F_{1j}.F_{k1}\right]\left[F_{1k} + \left(1-\varepsilon_j\right)F_{1j}.F_{jk}\right]$

With reference to diagram 2 and by making use of equations (c58) and (c59) from appendix 4, we can write:

$$f_2 = F_{21} + \sum_{j=3}^{4} \frac{1}{1-CR}\left(Y_1 + Y_2 + Y_3 + Y_4 + \frac{O_0 + O_0' + G_0 + G_0'}{1\text{-}CR_9}\right) \quad (54)$$

Where, for $j = 3$, $k = 4$, and for $j = 4$, $k = 3$ and for the derivation of the values of Y_1, Y_2, Y_3, Y_4 O_0, O_0', G_0, G_0' and CR_9 see appendix 3.

$$O_0 = \frac{(1-\varepsilon_2)\left(1-\varepsilon_j\right)^2 .(1-\varepsilon_k)F_{2j}.F_{jk}.F_{k2}\left[F_{j1} + (1-\varepsilon_2)F_{j2}.F_{21}\right]\left[F_{2j} + \left(1-\varepsilon_j\right)F_{2k}.F_{kj}\right]}{\left[1-CR_7\right]\left[1-CR_8\right]} \quad (55)$$

$$O_0' = \frac{(1-\varepsilon_2)\left(1-\varepsilon_j\right)^2 .(1-\varepsilon_k)^2 F_{2j}.F_{jk}.F_{k2}.V_3}{\left[1-CR_7\right]\left[1-CR_8\right]^2} \quad (56)$$

$V_3 = \left[F_{k1} + (1-\varepsilon_2)F_{k2}.F_{21}\right]\left[F_{jk} + (1-\varepsilon_2)F_{j2}.F_{2k}\right]\left[F_{2j} + \left(1-\varepsilon_j\right)F_{2k}.F_{kj}\right]$

$$G_0 = \frac{(1-\varepsilon_2)\left(1-\varepsilon_j\right)(1-\varepsilon_k)F_{2j}.F_{j2}\left[F_{k1} + (1-\varepsilon_2)F_{k2}.F_{21}\right]\left[F_{2k} + \left(1-\varepsilon_j\right)F_{2j}.F_{jk}\right]}{\left[1-CR_7\right]\left[1-CR_8\right]} \quad (57)$$

$$G_0' = \frac{(1-\varepsilon_2)\left(1-\varepsilon_j\right)^2 .(1-\varepsilon_k)F_{2j}.F_{j2}.V_4}{\left[1-CR_7\right]^2.\left[1-CR_8\right]} \quad (58)$$

$V_4 = \left[F_{j1} + (1-\varepsilon_2)F_{j2}.F_{21}\right]\left[F_{kj} + (1-\varepsilon_2)F_{2j}.F_{k2}\right]\left[F_{2k} + \left(1-\varepsilon_j\right)F_{2j}.F_{jk}\right]$

With reference to diagram 3 and appendix 4 we can get:

$$f_3 = \sum_{j=3}^{4} \frac{1}{1-CR}\left(Z_1 + Z_2 + \frac{M_0 + M_0' + P_0 + P_0'}{1\text{-}CR_{10}}\right) \quad 9+ \quad (59)$$

Where, for j = 3, k = 4, and for j = 4, k = 3 and for the derivation of the values of Z_1, Z_2, M_0, M'_0, P_0, P'_0 and CR_{10} (see appendix 4).

$$M_0 = \frac{(1-\varepsilon_2)^2 \cdot \left(1-\varepsilon_j\right)^2 \cdot (1-\varepsilon_k) F_{2j} \cdot F_{jK} \cdot F_{K2} \cdot F_{j2} \cdot \left[F_{2j} + (1-\varepsilon_k) F_{2k} \cdot F_{kj}\right]}{[1-CR_7][1-CR_8]} \tag{60}$$

$$M'_0 = \frac{(1-\varepsilon_2)^2 \cdot \left(1-\varepsilon_j\right)^2 \cdot (1-\varepsilon_k)^2 \cdot F_{2j} \cdot F_{jk} \cdot F_{k2}^2 \cdot \left[F_{2j} + (1-\varepsilon_k) F_{2k} \cdot F_{kj}\right] \cdot \left[F_{jk} + (1-\varepsilon_2) F_{j2} \cdot F_{2k}\right]}{[1-CR_7][1-CR_8]^2} \tag{61}$$

$$P_0 = \frac{(1-\varepsilon_2)^2 \cdot \left(1-\varepsilon_j\right) \cdot (1-\varepsilon_k) F_{2j} \cdot F_{j2} \cdot F_{k2} \left[F_{2k} + \left(1-\varepsilon_j\right) F_{2j} \cdot F_{jk}\right]}{[1-CR_7][1-CR_8]} \tag{62}$$

$$P'_0 = \frac{(1-\varepsilon_2)^2 \cdot \left(1-\varepsilon_j\right)^2 \cdot (1-\varepsilon_k) F_{2j} \cdot F_{j2}^2 \left[F_{kj} + (1-\varepsilon_2) F_{2j} \cdot F_{k2}\right] \cdot \left[F_{2k} + \left(1-\varepsilon_j\right) F_{2j} \cdot F_{jk}\right]}{[1-CR_7]^2 \cdot [1-CR_8]} \tag{63}$$

3. DISCUSSION OF THE RESULTS

The network method calculates the total heat loss or gain per unit area, q_i, at any surface due to the presence of other surfaces. Whereas, the infinite reflections method calculates heat exchange between two surfaces $q_{(n)1 \leftrightarrow 2}$ due to the presence of other surfaces. So that and in order to be able to do comparison between the two methods of calculation, equation (8) and (46) has to be rearranged in the following way :

$$q_1 = q_{(n)1 \leftrightarrow 2} + q_{(n)1 \leftrightarrow 3} + \ldots\ldots$$

$$q_2 = q_{(n)2 \leftrightarrow 1} + q_{(n)2 \leftrightarrow 3} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$q_n = q_{(n)3 \leftrightarrow 1} + q_{(n)3 \leftrightarrow 2} + \ldots\ldots$$

q_1, q_2, …, and q_n are heat loss or gain at surfaces 1, 2, …., and n.

For the simplification of the comparison and the validation, we have selected the following cases:

3.1. Three surface interaction:

Consider three opaque grey-diffuse planar surfaces 1, 2 and 3 forming:

- an equilateral triangle shaped enclosure with equal areas $A_1 = A_2 = A_3$ (figure 8a)
- a right-angle-triangle-shaped enclosure with different surface areas $A_1 \neq A_2 \neq A_3$ (figure 8b) which are sufficiently long so that the end effects are negligible (figures 8c).

- The surfaces are opaque, grey and diffuse. Their emissivities are respectively; $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.3$, $\varepsilon_3 = 0.5$ and are maintained at temperatures $T_1 = 300$ K, $T_2 = 283$ K, $T_3 = 318$ K respectively.

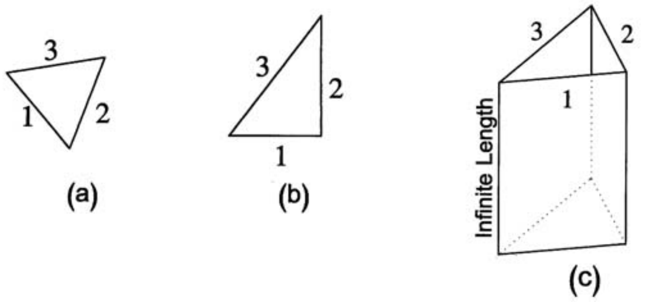


Fig. 8: A three surface enclosure infinitely long: Plane view of a right angle triangle shaped enclosure, (b) Plane view of an equilateral triangle shaped enclosure , (c)Volumetric view

For equilateral triangle enclosure with end effect negligible, $F_{12} = F_{21} = F_{13} = F_{31} = F_{32} = F_{23} = 0.5$. For a right-angle-triangle enclosure F_{12}, F_{21}, F_{13}, F_{31}, F_{32} and F_{23} are different and their values depend upon the areas A_1, A_2 and A_3 forming the enclosure. For planar surfaces $F_{11} = F_{22} = F_{33} = 0$

In this study we have chosen the following values: $A_1 = 3$ L, $A_2 = 4$ L, $A_3 = 5$ L , where L is the common length of the enclosure which is assumed long enough that the length of each side of the right angle triangle is negligible. Consequently the values of the configuration factors are; $F_{12} = 1/3$, $F_{13} = 2/3$, $F_{21} = 1/4$, $F_{23} = 3/4$, $F_{31} = 2/5$, $F_{32} = 3/5$. According to the N.W.M the net radiation heat fluxes per unit area q_i (i=1, 2 and 3) for each of the three surfaces is calculated from the governing matrix equation for the radiosities is written from equation (6). Substituting the numerical values into the matrix equation, and solving for J_1, J_2 and J_3 . Equation (8) is now used to determine the net radiation heat fluxes: $q_1 = \frac{\varepsilon_1}{1-\varepsilon_1}\left(\sigma.T_1^4 - J_1\right)$, $q_2 = \frac{\varepsilon_2}{1-\varepsilon_2}\left(\sigma.T_2^4 - J_2\right)$ and $q_3 = \frac{\varepsilon_3}{1-\varepsilon_3}\left(\sigma.T_3^4 - J_3\right)$

For an equilateral triangle shaped enclosure, the results are presented in table 1.

Table 1: Comparison of q (Wm^{-2}) values calculated by the I.R.M and the N.W.M for an equilateral shaped enclosure.

$q(Wm^{-2})$	N.W.M	I.R.M
q_1	-4.0	-4.0
q_2	-44.9	-44.9
q_3	48.9	48.9

For a right-angle triangle shaped enclosure, the results are presented in Table 2. It is clear from both tables that both I.R.M and N.W.M agree well.

Table 2: Comparison of q (Wm^{-2}) values calculated by the I.R.M and the N.W.M for a right-angle-triangle-shaped enclosure.

$q(Wm^{-2})$	N.W.M	I.R.M
q_1	-5.84	-5.84
q_2	-49.96	-49.96
q_3	43.47	43.47

If we reduce considerably the emissivity values to become $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.001$ we will also get, for the case of a right-angle triangle shaped enclosure, agreement in the results of both methods as can be seen Table 3.

Table 3: Comparison of q (Wm^{-2}) values calculated by the I.R.M and the N.W.M for a right-angle-triangle-shaped enclosure with $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.001$.

$q(Wm^{-2})$	N.W.M	I.R.M
q_1	-0.018	-0.018
q_2	-0.114	-0.114
q_3	0.102	0.102

But if one of the surfaces of the enclosure (here is selected an equilateral triangle shaped as example) has emissivity equal to one or to zero then the results will relatively disagree when calculating as shown in Tables 4 and 5. This disagreement comes from the fact that N.W.M has some limitations when the emissivity equals unity or zero.

Table 4: Comparison of q (Wm^{-2}) values calculated by the I.R.M and the N.W.M for an equilateral shaped enclosure with $\varepsilon_3 = 1$.

$q(Wm^{-2})$	N.W.M	I.R.M
q_1	-7.956	-7.956
q_2	-57.498	-57.498
q_3	Undetermined	65.454

3.2. Four surfaces interaction:

For more validation of the I.R.M, we have selected this time a rectangular-shaped enclosure (see figure 9). The enclosures are sufficiently long in the direction perpendicular to the planes of the figures so that the end effect can be neglected. The surfaces are opaque, grey and diffuse. Their emissivities are respectively; $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.2$, $\varepsilon_3 = 0.3$, $\varepsilon_4 = 0.5$ and are maintained at temperatures $T_1 = 300$ K, $T_2 = 283$ K, $T_3 = 318$ K and $T_3 = 290$K respectively.

Table 5: Comparison of q (Wm^{-2}) values calculated by the I.R.M and the N.W.M for an equilateral shaped enclosure with $\varepsilon_3 = 0$.

$q(Wm^{-2})$	N.W.M	I.R.M
q_1	Undetermined	7.546
q_2	Undetermined	-7.546
q_3	Undetermined	0

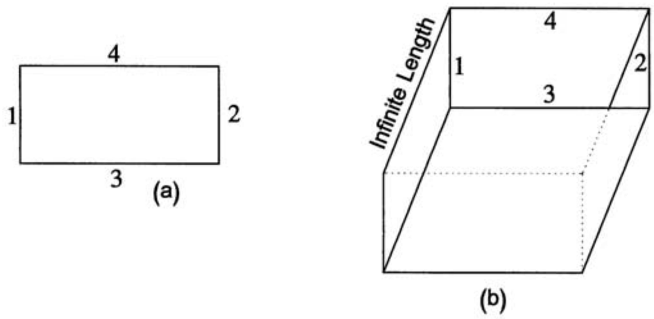


Fig. 9 : A Four surface enclosure infinitely long: (a) Plane view of a square-shaped enclosure, (b) Volumetric view

For a rectangular-shaped enclosure of dimensions (3 x 6 x L) with $A_1 = A_2 = 3$ x L, $A_3 = A_4 =$ 6 x L the appropriate configuration factors are; $F_{12} = F_{21} = 0.2361$, $F_{13} = F_{14} = F_{23} = F_{24} = 0.3819$, $F_{31} = F_{32} = F_{41} = F_{42} = 0.1909$, $F_{34} = F_{43} = 0.6180$.

It can be seen from Table 6 that N.W.M agrees well with I.R.M method for a rectangular duct. This confirms even more the validity of I.R.M as an exact method of calculation.

Table 6: Comparison of $q(Wm^{-2})$ (values calculated by I.R.M and the N.W.M for a rectangular-shaped enclosure.

$q(Wm^{-2})$	N.W.M	I.R.M
q_1	0.62	0.62
q_2	-18.62	-18.62
q_3	42.32	42.32
q_4	-33.31	-33.31

3.3. Two surface interaction

If we cconsider two finite surfaces 1 and 2 of areas $A_1$and A_2 where $A_1 = \frac{A_2}{2}$ as shown in figure 10. Surface 1 has emissivity $\varepsilon_1 = 0.1$ and is maintained at temperature $T_1 = 300$ K. Surface 2 has emissivity $\varepsilon_2 = 0.3$ and is maintained at temperature $T_2 = 283$ K. For a configuration factor $F_{12} = 0.2$ then $F_{21} = 0.1$. The use of I.R.M analysis suggests that equation (46) simplifies into equation (4) and can be used for the calculation. Equations (46), (8) and (1) are used for the comparison. Calculated values are presented in Table 7.

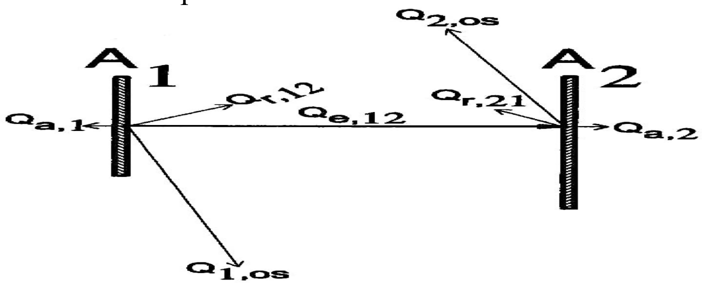


Fig. 10: Sketch for radiation exchange between two surfaces due to two-surface interaction

Table 7: Comparison of $q_1 = q_{(2)1 \leftrightarrow 2}$ (Wm^{-2}) values calculated by I.R.M and other methods for two surfaces with $\varepsilon_1 = 0.1$ and $\varepsilon_2 = 0.3$.

	I.R.M and Crabol Formula	N.W.M	Hottel's Formula
F_{12}=2 F_{21}=0.2	0.58	43.65	6.30
F_{12}=2 F_{21}= 1	7.75	7.75	7.75

It can be noticed from Table 7 that for view factors different from 1, I.R.M and Crabol formula agree well, whereas they disagree with both the N.W.M and Hottel's formula . The disagreement is a logical consequence of the fact that N.W.M and Hottel's formula are derived on the assumption that the surfaces form an enclosure, such as two infinitely large parallel plates, two coaxial long cylinders, or two concentric spheres, and that all radiation will be exchanged between the two surfaces and nothing else. Whereas in reality, the surfaces may not form an enclosure and thus some of the radiation will be transferred out of the system entirely. That is the reason why I.R.M gives less values than those produced by N.W.M and Hottel's equations. This is confirmed, however, by the fact that if we assume a two-surface enclosure that is when F_{12}=2 F_{21}= 1, I.R.M will in fact agree with N.W.M and Hottel's as you can see from the same table. If both surfaces have been given $\varepsilon_1 = \varepsilon_2 = 1$, thus $(1-\varepsilon_1)$ and $(1-\varepsilon_2)$ in equation (1) and (4) will be zero and thus we observe no difference in the results between I.R.M, Hottel and Crabol. Whereas the N.W.M will fail to produce a real value (see Table 8). The undetermined value resulted from N.W.M is due to the problem of denominator which contains $(1-\varepsilon_1)$ or $(1-\varepsilon_2) = 0$ (see equation (8)).

Table 8: Comparison of $q_1 = q_{(2)1 \leftrightarrow 2}$ (Wm^{-2}) values calculated by I.R.M and other methods For two surfaces with F_{12}=2 F_{21}= 0.2.

Emissivities	I.R.M and Crabol formula	N.W.M	Hottel's Formula
$\varepsilon_1 = \varepsilon_2 = 1$	19.12	Undetermined	19.12

3.4 Simplified formula:

For simplicity and practical purposes, one may generally neglect multiple reflections so that $(1-\varepsilon_1)$, $(1-\varepsilon_2)$, $(1-\varepsilon_3)$, $\ldots \approx 0$, without loss of much accuracy for high emissivity surfaces. So that; $B = f_1 = F_{12}$ and the exact equation (46) can be simplified into;

$$Q_{(n)1\leftrightarrow 2} = \varepsilon_1 \cdot \varepsilon_2 . \sigma . A_1 . \left(T_1^4 - T_2^4\right) \cdot F_{12} \tag{68}$$

The simplified formula (68) gives always lower values of heat flux in comparison to values given by the exact formula (46) as shown in Table 9.

Table 9: Comparison of $q_{(n)1\leftrightarrow 2}$ (Wm^{-2}) values calculated by the Exact formula and the Simplified one for high emissivity surfaces of an enclosure.

Emissivity $\varepsilon_1 = \varepsilon_2 = \cdots \varepsilon_4$	$q_{(n)1\leftrightarrow 2}$ (Wm^{-2})	Exact formula (46)	IS formula (69)	Difference (%)
0.7 0.8 0.9	$q_{(3)1\leftrightarrow 2}$ for an equilateral triangle - shaped enclosure	29.09 34.76 40.96	23.42 30.57 38.71	-19.49 -12.05 -5.49
0.7 0.8 0.9	$q_{(4)1\leftrightarrow 2}$ for a square-shaped enclosure	20.27 23.89 27.45	15.62 20.39 25.81	-22.94 -14.65 -5.97
0.7 0.8 0.9	$q_{(4)1\leftrightarrow 2}$ for a rectangular-shaped enclosure	14.13 16.75 19.55	11.06 14.44 18.28	-21.73 -13.79 -6.50

If the emissivity is 0.7, simplified formula (SF) gives around 20 % lower values than the values given by the exact formula (EF). Hence, SF can be improved in accuracy, to about -5% lower than EF, by adding to it the reduced percentage (RP) so that: Improved SF or ISF = SF $+|RP|$.SF = 1.2 SF $\approx$ EF.

Example, from Table 9 we can see that for emissivity 0.7, $|RP| \approx 20\%$ as an average for three and four-surface interaction. For SF = 23.42, RP = -19.49 % so $|RP| \approx$ 20 %, and $|RP|$.SF = 4.68. Thus ISF = 23.42 + 4.68 = 28.10 $\approx$ Exact Formula with only -3.4 % error. From the same table for SF = 15.62, RP =22.94 % so $|RP|$ $\approx$ 20 %, and RP x SF = 18.74 $\approx$ EF with only -7.5 % error.

For surface emissivity 0.8, RP has an average value of -15 % and so ISF = 1.15 SF. If the emissivity is 0.9, the average value for RP is -5 % and so ISF = 1.05 SF. For values of emissivity in between we can make an interpolation or a simple average. Hence, if we make use of equation (68), improved simplified formula (ISF) can be written as:

$$ISF = (1 + |RP|)\ Q_{(n)1\leftrightarrow 2} \tag{69}$$

4. CONCLUSION

An exact analysis called "Infinite Reflection Method" has been developed in order to calculate the radiation heat exchange between two diffuse grey-surfaces with the interaction of up to four surfaces. This contains newly derived factors which allow for the effect of multiple reflections at all interacting surfaces. As they were neglected by the traditional equation (1). The method is based upon two basic concepts; the arborescent diagrams and the infinite series algebra. Comparison of the results shows that values calculated by the I.R.M agree well with the values calculated by the N.W.M only for surfaces forming an enclosure and if their emissivities are not equal to 0 or 1. Hottel's formula is only valid for two surface enclosure and the use for more than two surfaces enclosure is misleading.

The advantages of I.R.M over the existing Methods are:

- it is valid for enclosures and non enclosures for any value of surface emissivity (no restriction when emissivity of a surface is 0 or 1),
- its simplicity for calculation because it does not requires computer and matrix algebra implementation as does the N.W.M. The factors f_1, f_2 and f_3 in equation (46) are only calculated once and allow this equation to be linearised for its direct use,
- it allows direct calculation of net radiation exchange between each pair of surfaces from which an effective radiation coefficient can be calculated. This simplifies linearisation of radiative heat transfer to be used with other modes of heat transfer. This is not possible when using N.W.M.

For high emissivity surfaces (ε = 0.7 to 1) improved simplified formula (69) can be used without loss of much accuracy. However, the I.R.M may have some limitations in that in some instances an analysis assuming diffuse-grey surfaces cannot yield good results. For example, if the temperatures of the individual surfaces differ considerably from each other, then a surface will be emitting predominantly in the range of wavelengths characteristic of its temperature while receiving energy predominantly in a different wavelength region. The I.R.M may not be valid in case of interference of external radiation.

NOMENCLATURE

A Area of the surface, (m^2)

CR Common ratio

F View factor

Q_i Heat loss or gain at surface i, (W)

T Absolute temperature, (K)

h_{r12} Radiation heat transfer coefficient between surface 1 and 2, ($Wm^{-2}K^{-1}$)

q_i Heat loss or gain per unit area at surface i, ($W\,m^{-2}$)

$q_{(n)i \leftrightarrow j}$ Net radiation heat exchange per unit area between surface 1 and 2 due to the interaction of n surfaces, Wm^{-2}

Q_{ij} Fraction of radiation leaving surface i which arrives at surface j, (W)

$Q_{(n)i \leftrightarrow j}$ Net radiation heat exchange between surface 1 and 2 due to the interaction of n surfaces(W)

Greek symbols

ε Emissivity of the surface

α Absorptivity of the surface

σ Stefan-Boltzmann's constant equals to $5.67\ 10^{-8}$ ($Wm^{-2}K^{-1}$)

REFERENCE

[1] H. C. Hottel "*Notes on Radiant Heat Transmission*", Chem. Eng. Dept., M.I.T. (1951)

[2] H. C. Hottel and A. F. Sarofim "Radiative Transfer", McGraw-Hill Book Company, New York (1967)

[3] A. K. Oppenheim "*Radiation Analysis by the Network Method*", Trans. ASME 78 (1956) 725-735.

[4] J. Crabol "Transfert de chaleur: applications industrielles", tome 2, Masson, Paris (1990)

[5] J. P. Holman "Heat Transfer", McGraw-Hill Book Company , London (1990)

[6] M.N. Ozisik "Heat transfer: a basic approach", McGraw-Hill Book Co., New York (1985)

[7] J. F. Sacadura "Initiation aux Transferts Thermiques", Technique et Documentation, Paris (1993)

[8] J. R. Howell "A catalogue of radiation configuration factors", McGraw-Hill Book Co., New York (1982)

[9] J. A. Clark "Energy simulation in building design", Adam Higher Ltd. Bristol (1985)

[10] F. Kreith "Principles of Heat Transfer", Intext Educational Publishers, New York (1973)

[11] A. Bouchair "*External environmental temperature : Proposed new formulation*", Building Services Engineering Research and Technology 22 (3) (2001) 133-156.