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Modelling and Simulation of Water Supply Systems for Feedback Control

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Abstract – *Operation principles of water supply systems are considered and the mathematical expressions for the system major components such as pipes, reservoirs and pumps are represented. Behaviour of thesystem is investigated by slow movement of fluid flow around steady-state operating point. Head losses changing nonlinearly with water flowrates in pipes are represented by the Darcy-Weisbach and the HazenWilliams methods with roughness dependent friction coefficients. Pumps are represented with second ordernonlinear function depending on their speed and pumped water flowrate. The city of Gaziantep Water SupplySystem is considered in the Case Study. The original pump characteristics obtained from the pump manufacturer are used for the mathematical equivalent of the pumps. Roughness dependent friction factors are calculated for different methods using bisection numerical solution method and the results are given andcompared. The major losses obtained with the Darcy-Weisbach and the Hazen-Williams methods are presented and compared with local losses. The simulation results and real plant measured data are compared.*

Résumé – *Les principales opération des systèmes d'alimentation en eau sont considères et les modèles mathématiques des éléments essentiels du système tels que les pipes, réservoirs et les pompes sont représentes. L'évaluation du comportement du système a été effectue grâce au faible mouvement du flux du liquide autour du point stationnaire d'exploitation. les pertes principales changeant non-linéairement avec des débits d'eau dans des pipes sont représentées par les méthodes de Darcy-Weisbach et Hazen-Williams en fonction des coefficients de friction a faible approximation. Les pompes sont représentées par des fonctions non linéaire du second ordre dépendant de leur vitesse et des débits d'eau pompee. Le système d'alimentation en eau de la ville de Gaziantep a été étudie ci-dessous. Les caractéristiques des pompes fournit par le fabricant ont été utilisées pour l'établissement du modèle mathématique équivalent des pompes. Les coefficients de friction a faible approximation sont calcules par différentes méthodes utilisant [bissection numérique solution méthode], les sont pressentes et compares. Les pertes principales generees par les méthodes de Darcy-Weisbach et Hazen-Williams sont présentées et comparées avec les pertes locales. Les résultats fournit par la simulation et les données réelles mesurées sur le cite ont été compares.*

Keywords: Water supply systems – Pipes – Reservoirs – Pumps – Mathematical expressions – Friction factors – Second order nonlinear function – Simulation.

1. INTRODUCTION

Water systems have been a common problem since the beginning of civilization [1]. The increase in population, the improvements in economical conditions and industrial developments have increased the use of water. These have necessitated the water supply systems to be built in technical disciplines [2]. Generally, water supply systems consist of four major components: Retention, supply, treatment and distribution systems [3]. In the supply systems, pumps, pipes, reservoirs and valves play important role in the fluid process dynamic behaviour [4].

History of water supply system and work about is very old [5]. Water was transported with simple supply systems to central locations at the hydraulic grade line. Such early systems did not distribute water to the individual users because pipes, made of wood, clay, lead or bamboo [6] were not available for significant pressure changes [7]. The earliest scientific contributions were provided by Archimedes in about 250 B.C.. Romans possessed some knowledge but this was not based on any quantitative law [8]. Practical operation of the water supply systems was performed through the construction of great systems in Egypt, in the Middle East and in China [5]. Significant developments had been made after the fourteenth century by Leonardo da Vinci and by Newton. Development of iron pipe together with the development of pumps made the water possible to deliver to the individual users in the mid-seventeenth century [6]. Work of Hagen and Bernoulli improved the knowledge about the water systems. Later, others provided some useful studies [5,8,9]. All such developments accelerated greatly about the water system problems to be studied on a quantitative basis. Weisbach, Darcy, Blasius, Chezy, Manning, and Osborne Reynolds provided critical contributions. Many technical societies, many governmental agencies and laboratories were founded in the twentieth century and these increased their efforts to the solution of practical water problems. Empirical solutions, linear and non-linear analysis, transient and statistical concepts and application of heat and mass transfer theories to the water systems have been studied [1,2].

In the present paper, major components of a water supply system which play important role in the operation are considered and their mathematical representations are given systematically. A real system, Gaziantep Water Supply System, is studied in the Case Study in obtaining models and simulations. The results are compared with real measured data.

2. MODEL OF SYSTEM HYDRAULICS

The variables such as pump speed, water flowrates, reservoir heads, water speeds represent deviations from the nominal steady-state operating values to obtain small signal behaviour. Steady and uniform flows are assumed for the water supply system and the variations around nominal operating values do not deteriorate this generality. The reasons are that in practical case, there will always be slight variations of velocity, heads, pressures and flowrates, but, if the average values are constant, the flow is considered to be steady. In addition, when a fluid flows past a solid boundary, there will be variations of velocity in the region close to the boundary. In that case, if the size and shape of the cross section of the upstream of fluid is constant, the flow can still be considered to be uniform [10]. The flow disturbances in water supply systems are common [7] and then, the dynamic water flowrate demand Q_d m^3s^{-1} from a supply system may be given:

$$Q_d = f(N, h_t, n, h_s, Q, h_{loss}) \tag{1}$$

where N denotes pump speed, n denotes number of pumps operating in parallel, h_t and h_s denote variable and static heads in reservoirs, Q is the water flowrate through and h_{loss} denotes head losses through a particular system.

Pumps: Head developed by the variable-speed pumps varies nonlinearly with their speed N *rpm* and output water flowrate [11] Q_p m^3s^{-1}:

$$h_p(N, Q_p) = A_o N^2 + \frac{B_o}{n} N Q_p - \frac{C_o}{n^2} Q_p^2 \tag{2}$$

where A_o, B_o, C_o are the constants for a particular pump depending on component characteristics or can be calculated using the flowrate-speed characteristics obtained from a particular pump [11].

Pipes: Consider a pipe section with length l_p *m* and of area A_p m^2. If the head difference, *Dh* between two ends of a pipe section is considered, the following differential equation can be given including head loss [11]:

$$\frac{dQ(t)}{dt} = \frac{gA_p}{l_p}\left(Dh - h_{loss}(t)\right) \tag{3}$$

The flowrate and head loss may be given as:

$$\begin{cases} \mathrm{h_{loss}(t) = h_{loss}^{o} + \Delta h_{loss}(t)} \\ \mathrm{Q(t) = Q^{o} + \Delta Q(t)} \end{cases} \tag{4}$$

where $(.)^o$ denotes steady-state value and $Dh_{loss}(t)$ designates variable head loss caused by the variable water flowrate $DQ(t)$. There are several equations derived from theoretical considerations and experimental data for the friction loss in pipes [5,7,12,13]. The total loss in a pipe section may be given as:

$$h_{loss}(t) = h_{loss-f_p}(t) + h_{loss-l}(t) \tag{5}$$

$$\begin{aligned} h_{loss-l}(t) = {} & \sum_{i=1}^{x} h_{loss-i}(q_i, Q(t), f_p(e), r) + \sum_{j=1}^{y} h_{loss-j}(Q(t)) \\ & + \sum_{x=1}^{z} h_{loss-x}(Q_{a1}(t), Q_{a2}(t)) + \sum_{z=1}^{w} h_{loss-z}(Q_1(t), Q_2(t), r_c) \\ & + h_{loss-ent}(Q(t)) + h_{loss-exit}(Q(t)) \end{aligned} \tag{5a}$$

$h_{loss\text{-}fp}$ denotes friction loss, $h_{loss\text{-}l}$ denotes local losses, x denotes the number of bends, q_i denotes the angle of bending at i^{th} bend, y, z, w denote number of valves, enlargements and contractions, respectively. $Q_1(t)$ and $Q_2(t)$ denote flowrates before and after contraction, $Q_{a1}(t)$ and $Q_{a2}(t)$ denote flowrates before and after enlargements, r_c denotes ratio of diameters of the two pipes and f_p denotes friction coefficient.

The Hazen-Williams approach is frequently used in head loss calculations and has a nonlinear characteristics with the water flowrate, $Q(t)$ as [14]:

$$h_{loss,f_p}(t) = \left(10.78 l_p)/(c^{1.852} D^{4.87}\right) Q^{1.852}(t) \tag{6}$$

where the constant 'c' denotes the pipe coefficient which depends on age, type of material used. D is the inner diameter of pipe in meter. Friction loss in pipes can also be calculated using the Darcy-Weisbach approach [15]:

$$h_{loss,f_p}(t) = (f_p l_p \big/ D A_p^2 2g) Q^2(t) \tag{7}$$

where g is the gravitational constant.

Local losses resulting from rapid changes in the direction or magnitude of the velocity of water [4] can be negligible for the long pipes of which length is higher than 100 diameter [8] but these are quite important for the small pipe sections [5]. Expansions, contractions and bends, valves, flow at entrance and flow at exit may cause local losses in piping process which can be expressed as a multiple of the square of flowrate in the pipe configuration [6]:

$$h_{loss-l}(t) = (K_m \big/ 2g\, A_p^2) Q^2(t) \tag{8}$$

where K_m is the local loss coefficient which depends on the physical conditions of the problem [6].

Friction Coefficient for Pipes There are several methods developed to calculate friction coefficient f_p for different types of pipes and flows [8,11,16,17]. The use of friction coefficient which depends on surface roughness can give more accurate results [10]. The Colebrook and White's empirical correlation is the best for this purpose [11]:

$$\frac{1}{\sqrt{f_p}} = -4 \log_{10}\left(\frac{e/D}{3.71} + \frac{2.51}{2\sqrt{2 f_p}\, N_R}\right) \tag{9}$$

where 'e' denotes the roughness of the pipe. Equation (9) should be solved graphically or using numerical methods to obtain more accurate results.

Water Reservoirs When a reservoir discharges under its own head without external pressure, the continuity equation [18] can be applied as:

$$\frac{d(rV(t))}{dt} = r_i Q_i(t) - r_o Q_o(t) \tag{10}$$

where r, r_i, r_o represent the water densities inside the reservoir, inflow and outflow respectively which are assumed constant and equal to each ($r = r_i = r_o = 1$). Q_i $m^3 s^{-1}$ and Q_o $m^3 s^{-1}$ denote reservoir input and output water flowrates, respectively and V m^3 designates volume of the water reservoirs.

3. CASE STUDY

The water is taken from Kartalkaya Dam which is 53 km. away from the city of Gaziantep whose block diagram is shown in Figure 1. h_{ti}, h_{si}, l_{pi} denote the variable heads (m) in the reservoirs, static heads (m), and the lengths of pipes (m), respectively. Q_a, Q_b, and Q_c designate the water flowrates through pipes. It is assumed that the water has uniform density in the pipe. The pipelines are buried underground, free of chemical reaction, biochemical, thermal and noise pollution and reverse flow would not occur. Check valves are fitted on the discharge side of the pumps to maintain forward flow in the main and to prevent back flow [19]. The system does not include cavitation. The air relief valves and pressure regulating valves are also used on the piping process. There are three pump stations and three reservoirs along the supply system. It is also assumed that the pumps in the first pump station have deviations around its nominal speed to obtain variations in fluid flowrate and heads in the reservoirs. Pumps are often stable because they handle incompressible fluids [20]. Three pumps work in parallel in the pump stations with the speed of 985 *rpm*. Each pump is equipped with a check valve at the delivery end for back flow prevention. The pipes used in the system are concrete type with an inner diameter of 1.4 meters and 15 years old. All of the reservoirs have cross sectional area of 475 m^2. Bending curvatures measured are larger than the pipe diameter, *(r/D)>4* in this case the coefficient K_m is taken 0.08 and the loss coefficient of check valves are taken K_m=3 [21]. Table 1 shows the numerical values about the water supply system of the city of Gaziantep.

Table 1: Numerical values of the process variables

$l_{p1}=669.27\ m.$	$A_{pi}=1.5394\ m^2.$	$D=1.4\ m.$
$l_{p2}=13805.04\ m.$	$h_{s1}=113.4\ m$	$g=9.81\ m/s^2$
$l_{p3}=20094.69\ m.$	$h_{s2}=210.4\ m.$	$r/D>4$
$l_{p4}=4689.04\ m$	$h_{s3}=283.4\ m.$	$N^o=985\ rpm.$
$A_t=475\ m^2.$	$h_{s4}=279.7\ m.$	Q^o@$2.8\ m^3/s.$
$r_i=r=r_o=1\ gr/cm^3.$		

The system is linearized around steady-state operating point (Q^o@$2.8\ m^3/s.,\ N^o=985\ rpm.$). The pump relation using characteristic curves provided by the pump manufacturer company (SMS A.S.) is calculated as:

$$h_p = 0.0001433N^2 + 0.005015NQ_p - 3.98Q_p^2 \tag{11}$$

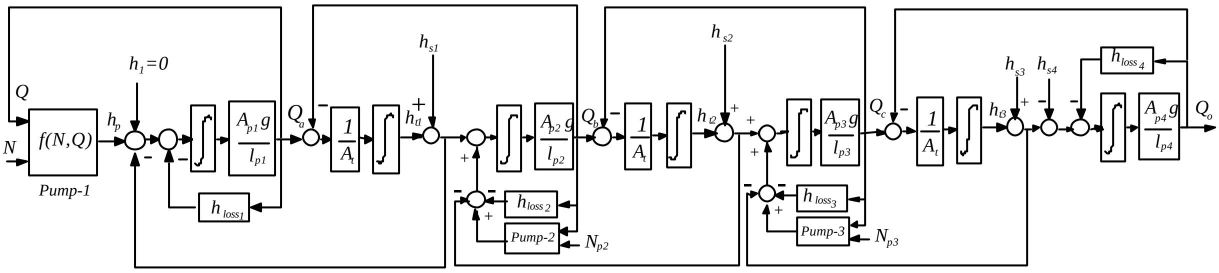

Fig.1: Block diagram of the water supply system

The supply system can be represented in state space matrix form such that the levels and flowrates can be considered as states. The input variable is the pump speed *N*, and the output variable is the water flowrate $Q_o(t)$. The canonical state space form is:

$$\dot{x}(t) = Ax(t) + Bu(t), \qquad y(t) = Cx(t) \tag{12}$$

where *x(t)* is the state matrix, *A, B, C* are the constant system matrices, *u(t)* is the input and *y(t)* is the system output. The state matrix *x(t)*, the input *u(t)* and the constant matrices, *A, B, C* are:

$$x(t) = \begin{bmatrix} Q_o & h_{t3} & Q_c & h_{t2} & Q_b & h_{t1} & Q_a \end{bmatrix}^T \tag{13}$$

$$u(t)=N \tag{14}$$

$$A = \begin{bmatrix} -0.0253 & 0.0032 & 0 & 0 & 0 & 0 & 0 \\ -0.0021 & 0 & 0.0021 & 0 & 0 & 0 & 0 \\ 0 & -0.0008 & -0.0398 & 0.0008 & 0 & 0 & 0 \\ 0 & 0 & -0.0021 & 0 & 0.0021 & 0 & 0 \\ 0 & 0 & 0 & -0.0011 & -0.0465 & 0.0011 & 0 \\ 0 & 0 & 0 & 0 & -0.0021 & 0 & 0.0021 \\ 0 & 0 & 0 & 0 & 0 & -0.0226 & -0.4553 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0.0067 \end{bmatrix}^T$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The system is 7th order and Table 2 shows the zeros and the poles of the system that is minimum phase and stable. Last three poles are near to the origin and would cause to slow system response.

Table 2: Zeros and poles of the water supplysystem

Zeros	Poles
-0.0396805	-0.4551564
-0.0255868	-0.0396796
-0.0250794	-0.0255573
-0.0002201±0.0000598i	-0.0250177
	-0.0003240
	-0.0002504
	-0.0000621

System Response Numerical calculations and simulations have been performed by integrating the governing differential equations. The response, output flowrate (Q_o) responding slowly is obtained by applying 985±10 *rpm* speed variation to the first pump as shown in Figure 2. The time constant of the system is about 7.00 hours and the steady-state water flowrate is about 2.8 m^3s^{-1}.

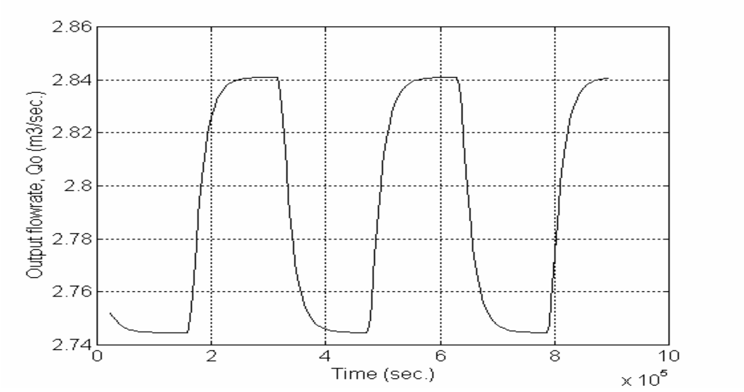

Fig. 2: Output flowrate $Q_o(t)$ for 985±10 *rpm* speed variation.

Figure 3 shows the effect of local and friction losses through the pipe section 3, l_{p3}. The local losses are very small in magnitude (≈0.8 m.) while the pipe friction losses are quite large and significant (≈43 *m.*) and dominant. This shows that friction loss for long pipes is important.

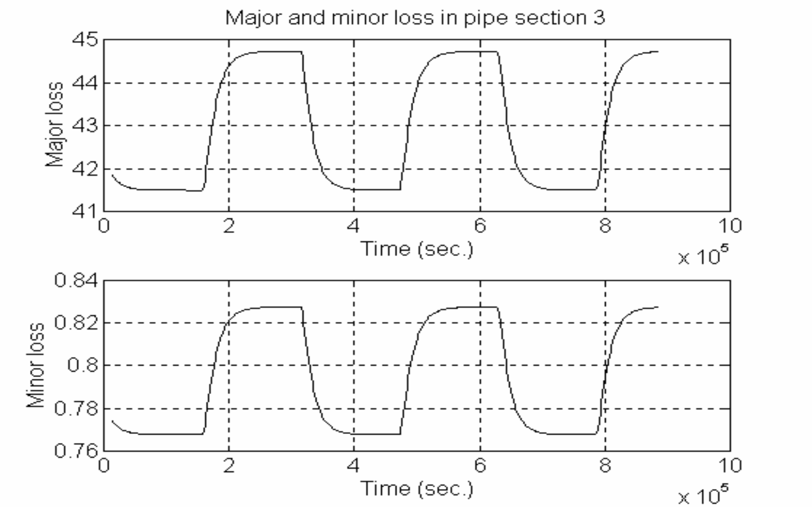

Fig. 3: Major and minor losses in pipe section 3.

Table 3 shows simulated and measured reservoir levels which are very close to the each other.

Table 3: Zeros and poles of the water supply

Reservoirs No	Measured heads (m)	Simulation results (m)
1	4.2	4.15
2	0.85	0.8
3	3.8	3.75

Friction Methods: Figure 4 shows the friction losses for the Hazen-Williams (42.8 m for the steady-state) and Darcy-Weisbach (43.1 m for the steady-state) methods, respectively. The magnitude of the loss simulated with the Darcy-Weisbach method (solid line) is greater than the other (dotted line).

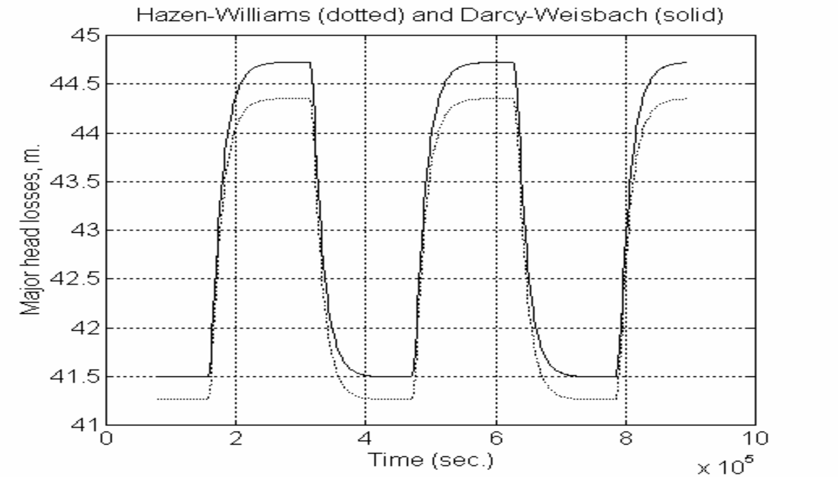

Fig. 4: Head losses for the Hazen-Williams and Darcy-Weisbach methods for the pipe section 3.

Friction Coefficient: The implicit friction coefficient given in Equation (9), f_p is calculated with a software program to run in Matlab environment using bisection numerical solution method [22]. The results are given in Table 4. It has been known that the Davies and White, the Prantdl and von Karman and the Blasius methods have been developed for the smooth pipes, that is why the calculated friction coefficients for these methods are different and smaller than that for the other methods. The results obtained from the other methods are very close to each other. The Colebrook and White method has been used in the simulations since it is the best for this purpose [11].

Table 4: Calculated friction coefficient for different methods.

Methods	f_p (ε)
Davies and White	0.00873830
Nikuradse	0.01810300
Gainguillet and Kutter	0.01754690
The Prantdl and von Karman	0.01055540
Colebrook and White	0.01833740
Moody	0.01901630
Blasius	0.00862908
Colebrook	0.01834350

4. CONCLUSIONS

The water supply systems have been studied such that the assessment of the water systems and the historical developments have been superficially explained. The mathematical equivalents of the water supply system components for small signal stability studies have been represented for uniform and steady flow conditions. The system has been considered deviating around steady-state operating point. The models given are to obtain the behaviour of the local real system. The reservoirs have been represented using the continuity equation. Hazen-Williams and Darcy-Weisbach methods have been used to represent the pipe friction losses. The non-linear relations have been derived for the pumps depending on the pump output flowrate and the speed of the pumps. The roughness dependent pipe friction coefficient has been taken into account and different methods are used to calculate, and the methods for rough pipes gave similar results. The mathematical equivalents represent low and midfrequency behaviour of the system.

A real industrial water supply system, the city of Gaziantep water supply system, has been introduced and the features of the system have been given in the Case Study. The numerical data about the system are obtained from the system operators. The simulation results which have been discussed with the plant operators and with the plant director coincide with the real system operation characteristics. The pipe headloss methods, the Darcy-Weisbach and the Hazen-Williams, have also been compared and it was shown that pipe friction is important and has significant effect for the long pipe sections. The software made to run in Matlab is used for the calculation of roughness dependent friction coefficient and Simulink toolbox is used for the simulations of the case study.

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