

Characterisation of Wings with NACA 0012 Airfoils

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Abstract – In this paper, we present a comparison of the experimental results of the behaviour of wings with elliptical, rectangular or trapezoidal plan forms with NACA 0012 airfoils. The prediction for the lift and the induced drag by the Prandtl lifting line theory is also compared with the experimental results. It appears that, at different Reynolds numbers, the aerodynamic characteristic differences between the three wings are very small.

Résumé – Dans cet article, on présente une comparaison des résultats expérimentaux des ailes de formes elliptiques, rectangulaires ou trapézoïdales de profils NACA 0012. La prédiction de la portance et la traînée induite par la théorie de la ligne portante de Prandtl est aussi comparée aux résultats expérimentaux. Il paraît qu'à de différents nombres de Reynolds, la différence des caractéristiques aérodynamiques entre les trois ailes est très réduite.

Key words: Prandtl lifting line theory – Aerodynamic characteristics – Lift – Induce drag – Fourier series – Finite wing – Incompressible flow – NACA 0012.

1. INTRODUCTION

The aerodynamic of airfoils has been studied by Kutta (1902) on thin airfoils and Joukowski (1905) on airfoils with thickness. These airfoils were obtained from a circular cylinder by conformal mapping. The theory of Joukowski for 2D flow, so for an infinite wing, marks the beginning of the modern aerodynamic. At the same time, appeared the works of pioneers of the aviation, the Wright brothers, who based their study on the analysis of the stability of the plane made by Lanchester in 1894 [1,4]. They had a good understanding of wing and airfoil behaviour well beyond that of other experimenters at that time. Most of the success of the Wright brothers could be attributed to their own research, which used their wind tunnel and numerous experiments with controlled kites and gliders. In the same way, the experimental studies take a big progress in the wind tunnel thanks to tests on models made in England by Wenham and Philips. A new step was crossed by Prandtl [3,12,19] who created the theory of finite wing (1917-1918) that was presented with a laborious mathematical formula that honours nowadays the domain of aerodynamics. This theory gave a new and a big progress to aerodynamics. During twenty years, the theories of Joukowski and Prandtl have been the starting points of many theoretical and experimental works, and permit us in this context to describe the aerodynamic properties of the elliptical wing [5,6] and of arbitrary plan form. This elliptical wing leads to an also elliptical distribution for the local circulation and to the famous minimum induce drag [13-19]. Generally speaking, one can represent any arbitrary distribution for local circulation by means of the Fourier series development. When the distribution of local circulation is not elliptical, the induce drag is not minimal but it appears that the difference is relatively small.

A geometric definition of wing airfoils is given by the figure 1[19,22,24]

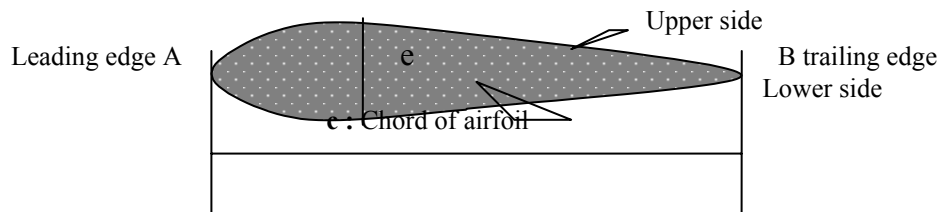


Fig. 1: geometry of the airfoil

2. AERODYNAMIC CHARACTERISTICS OF WINGS WITH ARBITRARY PLAN FORM

The starting point of the Prandtl lifting line theory is the Joukowski theorem in 2D flow giving the lift of a portion ℓ of the infinite wing, with the definition of the lift coefficient written in 2D for convenience $C_{L_{2D}}$, the surface of reference being the product $S = \ell c$:

$$L_{2D} = \rho V_{\infty} \ell \Gamma_{2D} = \frac{1}{2} \rho V_{\infty}^2 (\ell c) C_{L_{2D}} \quad \text{so that} \quad \Gamma_{2D} = \frac{1}{2} V_{\infty} c C_{L_{2D}} \quad (1)$$

If we use the classical result obtained in inviscid incompressible flow, the circulation is expressed by means of the angle of attack, the zero lift being determined by the Kutta-Koukowski condition at the trailing edge. This leads to the relation:

$$\Gamma_{2D} = 4\pi \bar{R} c V_\infty \sin(\alpha - \alpha_0) \quad (2)$$

where the ratio $\bar{R} = \frac{R}{c}$ is obtained either by numerical method or, if available, by the conformal mapping of the circular cylinder of radius R into our airfoil of chord c . Then (1) leads to the classical 2D result :

$$C_{L2D} = 8\pi \bar{R} \sin(\alpha - \alpha_0) = m_0 \sin(\alpha - \alpha_0)$$

where m_0 is the lift gradient at zero lift. It can be reminded that if the airfoil is considered as a thin airfoil then m_0 is equal to 2π . With this new notation, relation (2) can be written :

$$\Gamma_{2D} = \frac{m_0}{2} c V_\infty \sin(\alpha - \alpha_0) \quad (3)$$

Following Prandtl's ideas, relation (3) is extended to 3D flows on a large aspect ratio wing by formally changing $\Gamma(y) = \frac{m_0(y)}{2} c(y) V_e(y) \sin(\alpha_e(y) - \alpha_0(y))$

where $\alpha_e(y)$ is an effective incidence taking into account the induced velocities of the wing vortex sheet and the eventual wing twist. $V_e(y)$ is the modulus of the effective velocity acting on the airfoil. The preceding relation is generally simplified in the following way :

- the order of approximation of the lifting line theory the effective velocity $V_e(y)$ is kept equal to V_∞ due to the fact that, for large aspect ratio wings, the vortex sheet induced velocities are small compared to V_∞ .
- the angles of incidence are supposed to be small so that the sine is approached by the angles themselves expressed in radian.

Then we get :

$$\forall y \in [-b/2, +b/2] \quad \Gamma(y) = \frac{m_0(y)}{2} c(y) V_\infty [\alpha_e(y) - \alpha_0(y)] \quad (4)$$

Where

$$\alpha_e(y) = \alpha + \Delta\alpha_v(y) - VP \int_{-b/2}^{+b/2} \frac{\Gamma'(y_0)}{4\pi V_\infty (y - y_0)} dy_0 \quad (5)$$

Where VP means the Cauchy's principal value of the integral. Relation (4) with the help of relation (5) is the famous Prandtl's integro - differential equation (PIDE) which gives the unknown law of circulation when all the other parameters are given :

- the incidence α of the wing, i.e. the angle of incidence of the airfoil at the root of the wing
- the law of chord $c(y)$
- the law of twist $\alpha_v(y)$
- the lift gradient $m_0(y)$ of the airfoils which composed the wing
- the zero lift angle of attack $\alpha_0(y)$ which may vary either due to the use of different airfoils, or for airfoils with or without flaps.

Among the methods which exist for solving the PIDE (4), one can use a Fourier expansion of the law of circulation. If we introduce the angle θ such as $y = \frac{b}{2} \cos\theta$ we go from one tip of the wing to the other

when θ varies from 0 to π , the root of the wing corresponding to $\theta = \frac{\pi}{2}$.

Due to the fact that the law of circulation is necessarily zero at each tip, the Fourier expansion only consists in sine terms such as :

$$\Gamma(\theta) = 2b V_\infty \sum_{n=1}^{\infty} A_n \sin(n\theta) \quad (6)$$

With this expansion the induced velocities can be expressed by means of the A_n coefficients in the following way :

$$VP \int_{-b/2}^{+b/2} \frac{\Gamma'(y_0)}{4\pi V_\infty (y - y_0)} dy_0 = \sum_{n=1}^{\infty} n A_n \frac{\sin(n\theta)}{\sin\theta}$$

So that the PIDE (4) takes the following form, for any values of $\theta \in [0, \pi]$:

$$\sum_{n=1}^{\infty} A_n \sin(n\theta) = \frac{m_0(\theta)}{4b} c(\theta) \left[\alpha + \Delta\alpha_v(\theta) - \sum_{n=1}^{\infty} n A_n \frac{\sin(n\theta)}{\sin\theta} - \alpha_0(\theta) \right] \quad (7)$$

With
$$\alpha_e(\theta) = \alpha + \Delta\alpha_v(\theta) - \sum_{n=1}^{\infty} n A_n \frac{\sin(n\theta)}{\sin\theta} \quad (8)$$

In the above form it clearly appears that the coefficients A_n are solution of an infinite linear system which can be split into three subsystems. If we write :

$$G(\theta) = \frac{m_0(\theta)}{4b} c(\theta) \quad \text{and} \quad F_n(\theta) = \sin(n\theta) \left[1 + n \frac{G(\theta)}{\sin\theta} \right]$$

The three subsystems are, for any values of $\theta \in [0, \pi]$:

- • the effect of unit angle of attack
$$\sum_{n=1}^{\infty} a_n F_n(\theta) = G(\theta) \quad (9)$$

- • the effect of the twist
$$\sum_{n=1}^{\infty} B_n F_n(\theta) = G(\theta) \Delta\alpha_v(\theta) \quad (10)$$

- • the effect of the zero lift angles of attack
$$\sum_{n=1}^{\infty} C_n F_n(\theta) = -G(\theta) \alpha_0(\theta) \quad (11)$$

And the global solution
$$A_n = \alpha a_n + B_n + C_n \quad (12)$$

It is important to underline that each subsystem only depends upon geometrical data or 2D aerodynamics characteristics, so that they can be solved once for all for a given wing. The angle of attack only appears on the global solution (12). It can be shown that lift and induce drag can be express by means of the coefficients A_n by :

$$C_L = \pi A a_1 \quad \text{and} \quad C_{D_i} = \pi A \sum_{n=1}^{\infty} n A_n^2 \quad (13)$$

In the following part, we assume that the wing is composed of the same airfoil in smooth conditions so that $m_0(y) = m_0 = \text{constant}$ and in the same way $\alpha_0(y) = \alpha_0 = \text{constant}$, and that the wing is untwisted so that $\alpha_v(y) = 0$. Then we get from (10) that $B_n = 0 \quad \forall n$ and $C_n = -\alpha_0 a_n \quad \forall n$, so that from (12) :

$$A_n = (\alpha - \alpha_0) a_n \quad (14)$$

where a_n verifies the PIDE, $\forall \theta \in [0, \pi]$:

$$\sum_{n=1}^{\infty} a_n F_n(\theta) = G(\theta) \quad (15)$$

with
$$G(\theta) = \frac{m_0}{4b} c(\theta) \quad (16)$$

and
$$F_n(\theta) = \sin(n\theta) \left[1 + n \frac{G(\theta)}{\sin\theta} \right] \quad (17)$$

In that case, lift and drag coefficients takes the simplified form :

$$C_L = \pi A a_1 (\alpha - \alpha_0) \quad (18)$$

and
$$C_{D_i} = \frac{C_L^2}{\pi A} \sum_{n=1}^{\infty} n \left(\frac{a_n}{a_1} \right)^2$$

This last relation is often written

$$C_{D_i} = \frac{C_L^2}{\pi A} (1 + \sigma) \quad (19)$$

with
$$\sigma = \sum_{n=2}^{\infty} n \left(\frac{a_n}{a_1} \right)^2 \quad (20)$$

3. THE AERODYNAMIC CHARACTERISTICS OF THE ELLIPTIC WINGS

An important case, if that of an elliptical distribution of local circulation for which $A_n = 0 \quad \forall n \geq 2$. For an untwisted wing, composed of the same airfoil, this case is obtained for an elliptical plan form. Effectively the

above relations lead to $a_n = 0 \quad \forall n = 2$ and the PIDE reduces to $a_1 \sin\theta = (1 - a_1) \frac{m_0}{4b} c(\theta)$ after replacing $G(\theta)$ and $F_1(\theta)$, which is of the form :

$$c(\theta) = c_0 \sin\theta = c_0 \sqrt{1 - \left(\frac{y}{b/2}\right)^2}.$$

The only coefficient a_1 can be determined using the aspect ratio $A = \frac{b^2}{S}$ and the surface of reference of the

$$\text{wing } S = \int_{-b/2}^{+b/2} c(y) dy = \frac{b}{2} \int_0^\pi c(\theta) \sin\theta d\theta.$$

It comes :

$$a_1 = \frac{\frac{m_0}{\pi A}}{1 + \frac{m_0}{\pi A}} \quad \text{so that} \quad \boxed{C_L = \frac{m_0 (\alpha - \alpha_0)}{1 + \frac{m_0}{\pi A}} = m (\alpha - \alpha_0)}$$

And as $\sigma = 0$, the induce drag is minimum with $\boxed{C_{D_i} = \frac{C_L^2}{\pi A}}$

The distribution of circulation is $\boxed{\Gamma(\theta) = \frac{2b V_\infty C_L}{\pi A} \sin\theta = \Gamma_s \sqrt{1 - \left(\frac{y}{b/2}\right)^2}}$ and the effective angle of attack are constant $\alpha_e(\theta) = \alpha - A_1$ or : $\alpha_e(\theta) - \alpha_0 = \frac{\alpha - \alpha_0}{1 + \frac{m_0}{\pi A}}$.

4. AERODYNAMIC CHARACTERISTICS OF UNTWISTED RECTANGULAR AND TRAPEZOIDAL WINGS

For a rectangular and trapezoidal wings, the chord $c(y) = c_0$ is a constant and the aspect ratio is simply $A = \frac{b}{c_0}$. In that case, the PIDE (15) becomes :

$$\sum_{n=1}^{\infty} a_n \sin(n\theta) \left[1 + \frac{n}{\sin\theta} \frac{m_0}{4A} \right] = \frac{m_0}{4A} \quad \forall \theta \in [0, \pi]$$

If for convenience we assume that the wing is composed of a thin airfoil, then the above relation turns to :

$$\sum_{n=1}^{\infty} a_n \sin(n\theta) \left[1 + \frac{n}{\sin\theta} \frac{\pi}{2A} \right] = \frac{\pi}{2A} \quad \forall \theta \in [0, \pi] \quad (21)$$

For a symmetrical wing, the flow is also symmetrical so that for any values of θ one must obtain $\sum_{n=1}^{\infty} [1 + (-1)^n] a_n \sin(n\theta) = 0$. It turns out that all the even coefficients a_{2n} are zero so that solving relation (21) for the values of θ taken in the complete interval $[0, \pi]$ is useless. Consequently, (21) is rewritten, on one hand only for the odd coefficients, and secondly only for the values of θ taken within the interval $[0, \pi/2]$:

$$\boxed{\sum_{n=1}^{\infty} a_{2n-1} \sin[(2n-1)\theta] \left[1 + \frac{2n-1}{\sin\theta} \frac{\pi}{2A} \right] = \frac{\pi}{2A}} \quad \forall \theta \in [0, \pi/2] \quad (22)$$

On a numerical point of view, the infinite linear system (22) is truncated to a finite one, including only N coefficients a_{2n-1} unknown. Consequently the equation (22) is written for N discrete values θ_j .

Writing $X_n = a_{2n-1}$, we have to solve $\boxed{\sum_{n=1}^{n=N} A_{jn} X_n = B_j} \quad \forall j = 1, N$.

The matrix to inverse is $A_{jn} = \sin[(2n-1)\theta_j] \left[1 + \frac{2n-1}{\sin\theta_j} \frac{\pi}{2A} \right]$ and the second member $B_j = \frac{\pi}{2A}$.

• If only four unknown coefficients are kept with the values $\theta_1 = \frac{\pi}{8}$, $\theta_2 = \frac{\pi}{6}$, $\theta_3 = \frac{\pi}{4}$ and $\theta_4 = \frac{\pi}{2}$, we obtain the following results, with $A = 8$: $a_1 = 0.24301$, $a_3 = 0.02823$, $a_5 = 0.00508$ and $a_7 = 0.00218$, which leads to the aerodynamic coefficients, drawn from (18), (19) and (20) : $C_L = 4.5806 (\alpha - \alpha_0)$
 $\sigma = 0.04322$ and $C_{D_i} = 1.16126 (\alpha - \alpha_0)^2$

It appears that with such few points, the results are too sensitive to the choice of the angles θ_j . To cure this problem it is necessary to keep more unknown.

• With $N = 40$, that is to say with a_1 to a_{79} , and $\theta_j = \frac{(j - 1)}{(N - 1)} \frac{\pi}{2}$, the following global results are the following :

$$C_L = 4.53042(\alpha - \alpha_0) \quad \sigma = 0.04829, \text{ and } C_{D_i} = 1.14145(\alpha - \alpha_0)^2, \text{ or } C_{D_i} = 0.055613 C_L^2$$

It is to be noticed that for the value $\theta_1 = 0$, which corresponds to the first line of the linear system, the PIDE is taken to the limit giving $A_{1n} = (2n - 1)^2$ and $B_1 = 1$. There is no changes in these results with five digits, if the number N of unknown is increased up to 60, or 80 or 100. In parallel to these results dealing with the rectangular wing, one can compute the results for the elliptical wing with the same aspect ratio $A = 8$, and $m_0 = 2\pi$. It comes :

$$[C_L]_{ell.} = 4.71239(\alpha - \alpha_0) \quad \sigma = 0 \text{ and } [C_{D_i}]_{ell.} = 1.17810(\alpha - \alpha_0)^2 \text{ or } [C_{D_i}]_{ell.} = 0.053052 [C_L]_{ell.}^2$$

The lift curve slope $\frac{\partial C_L}{\partial \alpha}$ is 3.86% less than the elliptical one. On an other hand, compared with the same lift coefficient, the induce drag coefficient of the rectangular wing is 4.83% greater than the elliptical one.

5. COMPARISON OF NUMERICAL AND EXPERIMENTAL RESULTS

The three wings tested are equipped with a NACA0012 airfoil, which is a symmetrical airfoil of 12% thickness. The three wings, mounted at the wall of the wind tunnel, with a special device to get free of the boundary layer on this wall, have the same half span $b/2 = 0.32$ m, the same aspect ratio $A = 8$, and consequently the same surface S . For both wings, the mean chord is $\bar{\ell} = 0.1067$ m and the Reynolds number base on this length and the wind tunnel velocity $V_8 = 26.5$ m/s, is roughly $Re \approx 1.9 \cdot 10^5$. In these conditions, at positive angles of attack, a laminar separation occurs closed to the leading edge which provokes a transition followed by a rapid turbulent reattachment, so, despite the relatively low Reynolds number, the flow is turbulent on the entire wing.

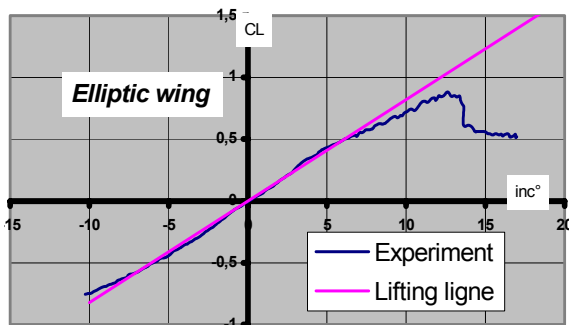


Fig. 2: Lift of rectangular wing

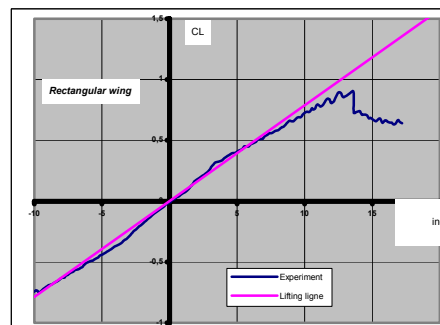


Fig. 3: lift of elliptic wing

a) Lift curves (Figures 2,3,7,8, 9)

The variation of the lift coefficient with the angle of attack α is linear up to approximately 6° . Then the progress of separation induces a decrease in the lift slope until the stall. On figure 2, the two experimental lift curves are superposed showing that the differences between the two wings are only visible near and after stall.

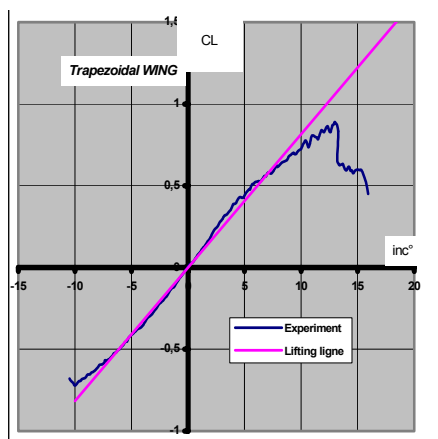


Fig. 4: Lift of trapezoidal wing

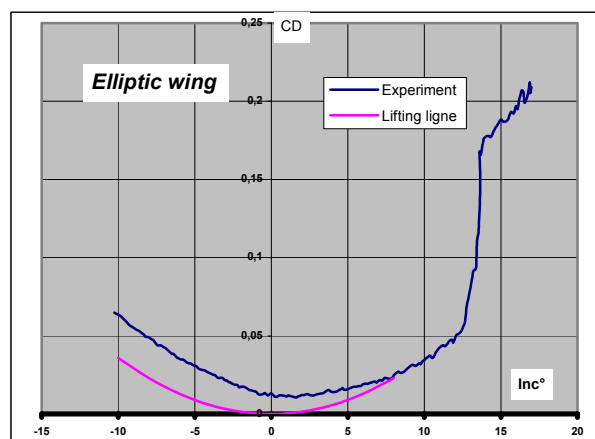


Fig. 5: Induced drag of elliptic wing

b) Drag curves (Figures 5,6,7)

The experimental and numerical curves are not immediately comparable due to viscous effects.

If we assume that the friction drag is not very sensitive to the angle of attack, then the experimental drag at zero lift, for which little separation is expected, gives a good idea of this part of the total drag. With this value added to the induce drag the remaining differences between $C_{D_0} = C_{D_0} + (1 + \sigma) \frac{C_L^2}{\pi A}$ and the experimental results come from flow separation and is visible only at angles of attack greater than 6° .

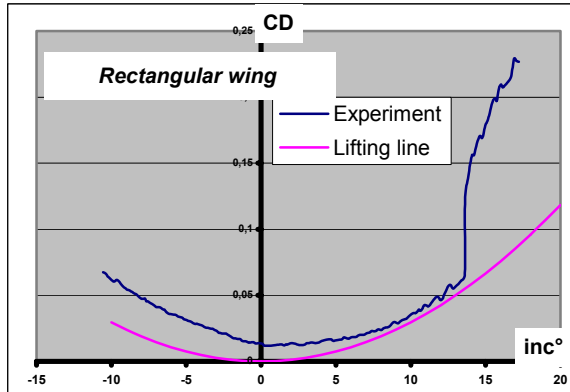


Fig. 6: Induced drag of rectangular wing

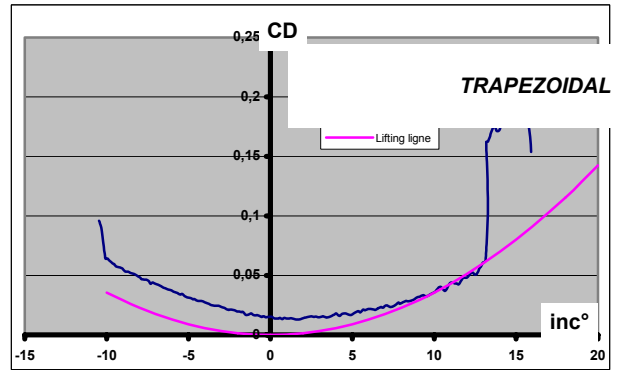


Fig. 7: Induced drag of trapezoidal wing

$CL = f (Inc)$

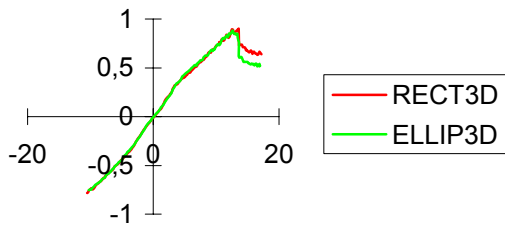


Fig. 8: Comparison between rectangular and elliptic wings given by experimental results.

$Cz = f (Inc)$

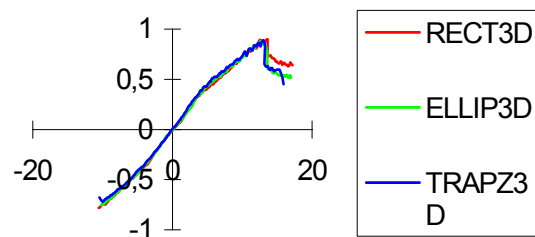


Fig. 9: Comparison between rectangular, elliptic and trapezoidal wings given by experimental results

6. CONCLUSIONS

In this paper, we have compared the aerodynamic characteristics of rectangular, trapezoidal and elliptic wings either on the experimental point of view or on the numerical one using the Prandtl lifting line theory. It appears that, to the Reynolds tested, the experimental behaviour of the two wings is very closed, with differences essentially near and after stall. The theoretical results predict reasonably well the lift slope as long as separations are of small importance, i.e. for small incidences. In the same spirit, once the friction drag is added to the theoretical induce drag, the resulting drag is also satisfactory for angles of attack with few separation.

It is possible to improve the lifting line theory results, but it is then necessary to take into account the experimental curve $C_{L_{2D}}(\alpha)$ including stall. This could be done starting from (1) and the lifting line principles giving the following relation :

$$\Gamma(y) = \frac{1}{2} V_\infty c(y) C_{L_{2D}}[\alpha_c(y)]$$

Due to the generally non linear behaviour of the function $C_{L_{2D}}(\alpha)$ in experimental conditions, the PIDE is no longer linear and must be solve by an iterative process.

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