

Satellite-Derived Surface Radiation Budget over the Algerian area. Estimation of Downward Solar Irradiance

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Abstract – *Downward surface solar irradiance received at ground, is an important component of the energy budget of ground surface. Indirect derivations of this parameter are common on the absence of direct measurements with reliable instruments. Our estimation is based on satellites observations. This paper presents development work on a tool for modelling the downward radiation received at ground. At first, this work is devoted to estimate the irradiance under all sky conditions from the geostationary weather satellite Meteosat-7. The model is based on NREL's SPECTRAL2 model for clear skies which takes no account of cloud attenuation of solar radiation. It can be shown to work well for its design purpose, but in maritime climates clear weather days are few and far between. Development work on the original model includes modification of the aerosol model and extended work on the Dedieu (1987) for developing a physically model to derive downward solar irradiance at the surface of the earth and surface albedo from Meteosat satellite measurements in the wavelength between 0.40 and 1.10 μm . The model takes into account Rayleigh and Mie scattering, water vapor and Ozone absorption. No threshold setting is necessary to distinguish between clear and cloudy conditions, there by avoiding the problem of its arbitrary nature and to some extent allowing quicker and easier data processing. On the other hand, a statistical method has been applied assuming a correlation between global horizontal surface irradiance under cloudless sky and a cloud index. The former is calculated using the cloudless spectral irradiance model (Heliosat method). The latter, i.e. cloud index, is derived from Meteosat-7 visible counts as a measure of cloud cover (Cano 1986) [1]. The Meteosat estimates of downward surface solar irradiance resulting from the two methods are compared with ground measurements.*

Résumé – *Malgré l'existence de réseau de mesures météorologiques, constitués d'appareils performants, ces derniers restent toujours insuffisants à l'échelle d'un pays ou même d'une région pour une bonne appréciation du potentiel énergétique. Cela a conduit, de nombreux auteurs, à développer des méthodes indirectes pour son estimation. Le rayonnement solaire, de part son caractère radiatif, est le processus le plus cohérent avec les méthodes d'acquisition en télédétection. Notre étude s'est focalisée sur le développement d'une méthodologie d'estimation de l'irradiation solaire qui atteint le sol à partir des données satellitales par le biais d'un modèle de transfert radiatif. Pour notre étude, nous nous sommes basé essentiellement sur les données numériques du satellite Meteosat (visibles). En premier, pour simuler les différentes valeurs du rayonnement au sol ainsi que les composantes qui atteignent le satellite, une méthode physique a été développée basée sur la méthode proposée par Dedieu (1987). Pour l'estimation des différentes transmittances un modèle physique double trajet s'est développé, basé sur le modèle Spectral2 proposé par Bird (1986). Pour la validation des résultats obtenus, nous avons appliqué une méthode statistique (Heliosat) basée sur un indice de nuage (Cano 1986) [1]. Les résultats obtenus par les deux méthodes sont comparés avec des mesures de terrain.*

Mots clés : Irradiation solaire – Meteosat 7 – Spectral2 – Heliosat.

1. INTRODUCTION

Solar energy, being the most known and the most famous energy appears today as the symbol of renewable energy. Beyond its direct impact on human being, solar radiation concerns several domains and applications, including for example: climatology, agrimeteorology, hydrology and solar energy converting system design for which solar radiation plays an important part in air-sea interactions; agro-meteorology and energy from biomass, because the growth of biomass directly depends on the photosynthesis and therefore on solar radiation; ageing of materials (such as polymers) that are damaged by UV radiations; and use of solar energy: solar water heating and photovoltaic, for which the dimensioning of PV cells should be improved to reduce investment costs. Thus a large number of people working in these domains (or in other fields not quoted above) need a precise knowledge of solar radiation and of its distribution in time and space. Direct measurements can be obtained from pyranometers. However, this network of these instruments is not always sufficiently dense, especially over the deserts, and the measurements are not always reliable in particular their intercalibration. Geostationary satellites offer an opportunity to overcome these, since they measure the radiation reflected by the earth-atmosphere system with space and time resolution suitable for most of the applications just mentioned.

2. METHODOLOGY

2.1. Estimation for shortwave radiation from Meteosat

2.1.1. Physical method

The global irradiance at ground level is determined by the atmospheric transmittance and the clearsky irradiance. While the clear sky irradiance is modelled with a site specific atmospheric condition. We have incorporated a parameterized spectral cloud free flux mode Spectral2 to compute the total, direct and diffuse irradiance. Spectral2 is a clear sky model which take no account of cloud attenuation of solar radiation. It can be shown to work well for its design purpose, but in maritime climates clear weather days are few and far between. For this, modification for cloudy skies is needed as both processes, cloud effect and atmospheric transmission are combined. In this estimation, we consider a cloudy scattering atmosphere with negligent absorption and molecular scattering assuming isotropy of the radiance reflected by the cloud layer and the earth's surface. The following radiative transfer model [6] is adopted for the solar irradiation estimation:

$$I = I_0 \varepsilon \cos \theta_s T(\theta_s) (1 - \alpha) (1 - \alpha_s) \quad (1)$$

Where I is the hourly solar irradiation (MJ/m^2), I_0 is the extraterrestrial irradiance, θ_s is the solar zenith angle, $T(\theta_s)$ is the transmittance of the atmosphere, α is the Planetary albedo, α_s is the earth's surface albedo and ε is the ratio of actual to mean sun-earth distance

The calculation of the transmittance of the atmosphere $T(\theta_s)$ is based on N'RELS SPECTRAL2 model for clear skies. This model corresponds to the spectral model proposed by Bird and Riordan (1986). The original model (Bird, 1984) is based on previous parametric models developed by Leckner (1978) [8] and by Brine and Iqbal (1983). Justus and Paris (1985) [4] and Bird and Riordan (1986) have made improvements to the simple model approached by Bird (1984) including refinements based on comparisons with measured spectra [7]. Under cloudless sky conditions, direct beam radiation constitutes the major part of the incoming solar shortwave radiation, above about 400 nm [2]. The model uses the extra terrestrial spectral irradiance obtained by Fröhlich and Wehrli of the World Radiation Center with a 10 nm resolution for 122 wavelengths in the range 300 nm to 4000 nm. The required input parameters are local geographic coordinates, atmospheric pressure, precipitable atmospheric water vapor and aerosol information. The aerosol information required by the model is the aerosol optical. The beam irradiance received at ground level by a surface normal to the sun's rays at wavelength λ under clear skies is modelled as the product of the extraterrestrial spectral radiation $I_{0n\lambda}$ and corrected for the actual sun-earth distance (Spencer, 1971) [12] from a series of independent optical processes that attenuate extra terrestrial solar radiation, given by:

$$I_{bn\lambda} = I_{0n\lambda} T_{R\lambda} T_{O\lambda} T_{g\lambda} T_{w\lambda} T_{a\lambda} \quad (2)$$

Where the other factors are the transmittances for the different extinction processes considered here: Rayleigh scattering, absorption by ozone, uniformly mixed gases and water vapor, and finally, aerosol extinction [9].

For all of these transmission processes except $T_{w\lambda}$ and $T_{g\lambda}$, the transmittance can be described by the classic Bouguer-Beer law:

$$T_{i\lambda} = \exp[-\tau_{i\lambda} m_i] \quad (3)$$

Where for each process i ($i = R$ gives $T_{R\lambda}$, etc), m_i is the optical mass and $\tau_{i\lambda}$ is the spectral optical depth. The earth-Sun distance factor as given by Spencer is [12]:

$$\varepsilon = 1.00011 + 0.034221 \cos \varphi + 0.00128 \sin \varphi + 0.000719 \cos 2\varphi + 0.000077 \sin 2\varphi \quad (4)$$

The day angle φ in radians is represented by:

$$\varphi = 2\pi(d - 1) / 365 \quad (5)$$

Air mass is a measure of the optical path length through the atmosphere and is determined by the zenith angle θ_s . The zenith angle is determined by the date, time, latitude, and longitude. The relative air mass as given by Kasten is:

$$m_r = [\cos \theta_s + 0.15(93.885 - \theta_s)^{-1.253}]^{-1} \quad (6)$$

The pressure-corrected air mass m_a is given by:

$$m_a = -(\cos \theta_s + 0.15(93.885 - \theta_s)^{-1.253})^{-1} \cdot P P_0^{-1} \quad (7)$$

Where: $P_0 = 1013.25$ Mb and P is measured surface pressure in mb.

The expression that we use for the atmospheric transmittance after Rayleigh scattering was taken from Kneizys et al. and is:

$$T_{R\lambda} = \exp(-m_a \tau_R) \quad (8)$$

Where

$$\tau_{R\lambda} = [115.6406\lambda^4 - 1.355\lambda^{-2}]^{-1} \quad (9)$$

The extinction coefficients used in the model are largely the same as those used in Spectral2. Turbidity is a difficult process to quantify as the particles can vary in size, in number, and in other subtle. However, it can be adequately described by an empirical relationship developed by Angstrom (1961) [3].

$$\tau_{a\lambda} = \beta \left(\frac{\lambda}{\lambda_1} \right)^{-\alpha}, \lambda_1 = 1\mu\text{m} \quad (10)$$

Where β is a measure of the amount of the turbidity and α characterises the particle size distribution.

The aerosol models include two average values of Angstrom's wavelength exponents α_1 and α_2 , for wavebands separated by $0.5\mu\text{m}$, respectively, thus angstrom's exponent α is the average value. The aerosol transmittance is then obtained as [2]:

$$T_{a\lambda} = \exp(-\beta\lambda^{-\alpha}m_a) \quad (11)$$

Where:

$$\begin{cases} \alpha = \alpha_1 & \text{if } \lambda < \lambda_0 \\ \alpha = \alpha_2 & \text{otherwise} \end{cases} \quad (12a)$$

and

$$\begin{cases} q = 2^{\alpha_2 - \alpha_1} & \text{if } \lambda < \lambda_0 \\ q = 1 & \text{otherwise} \end{cases} \quad (12b)$$

To measure turbidity, for this work, we permit the consideration of different aerosol standard models or the choice of a particular model defined by user. We adopted the water vapor transmittance expression of Leckner which has the form:

$$T_{w\lambda} = \exp \left[\frac{-0.2385a_{w\lambda} wM}{(1 + 20.07a_{w\lambda} wM)^{0.45}} \right] \quad (13)$$

Where w is the precipitable water vapor (cm) in a vertical path and $a_{w\lambda}$ is the water vapor absorption coefficient as a function of wavelength. For ozone, Leckner's transmittance equation was used, which is:

$$T_{o\lambda} = \exp(-a_{o\lambda} O_3 M_o) \quad (14)$$

Where $a_{o\lambda}$ is the ozone absorption coefficient, O_3 is the ozone amount (atm-cm) and M_o is the ozone mass. The ozone mass is given by:

$$M_o = \frac{1 + \frac{h_0}{6370}}{(\cos \theta_s^2 + 2 \frac{h_0}{6370})^{0.5}} \quad (15)$$

The parameter h_0 is the height of maximum ozone concentration, which is approximately 22 km. Leckner's expression for uniformly mixed gas transmittance is used, and it is expressed as:

$$T_{u\lambda} = \exp \left[\frac{-1.41a_{u\lambda} M'}{(1 + 118.3a_{u\lambda} M')^{0.45}} \right] \quad (16)$$

Where $a_{u\lambda}$ is the combination of an absorption coefficient and gaseous amount. We used Leckner's values of $a_{u\lambda}$. The diffuse irradiance on a horizontal surface is divided into three components: (1) the Rayleigh scattering component $I_{r\lambda}$, (2) the aerosol scattering component $I_{a\lambda}$, and (3) the component that accounts for multiple reflection of irradiance between the ground and the air $I_{g\lambda}$. The total scattered irradiance is then given by the sum [9]:

$$I_{d\lambda} = I_{r\lambda} + I_{a\lambda} + I_{g\lambda} \quad (17)$$

Where:

$$\begin{cases} I_{r\lambda} = I_{0\lambda} \varepsilon \cos \theta_s T_{o\lambda} T_{u\lambda} T_{w\lambda} T_{aa\lambda} (1 - T_{r\lambda}^{0.95})^{0.5} & (18) \\ I_{a\lambda} = I_{0\lambda} \varepsilon \cos \theta T_{o\lambda} T_{u\lambda} T_{w\lambda} T_{aa\lambda} T_{r\lambda}^{1.5} (1 - T_{as\lambda}) F_s C_s & (19) \\ I_{g\lambda} = (I_{d\lambda} \cos \theta_s + I_{r\lambda} + I_{a\lambda}) r_{s\lambda} r_{g\lambda} C_s / (1 - r_{s\lambda} r_{g\lambda}) & (20) \end{cases}$$

$$r_{s\lambda} = T_{o\lambda} T_{w\lambda} T_{aa\lambda} [0.5(1 - T_{r\lambda}') + (1 - F_s') T_{r\lambda}' (1 - T_{as\lambda}')] \quad (21)$$

$$T_{as\lambda} = \exp(-\omega_{\lambda} a \tau_{at} M_a) \quad (22)$$

The spectral global irradiance is represented by:

$$I_{g\lambda} = I_{bn\lambda} + I_{at} \quad (23)$$

The apparent ground albedo may be defined from a time series of Meteosat observations as follows: first, sensor outputs are converted into radiances. The radiance $L^i(i, j)$ detected by the satellite is a linear function of the numerical count and is calculated by:

$$L^i(i, j) = a(cn - cn_0) \quad (24)$$

Where cn is the satellite numerical count, cn_0 is the space count and a is the calibration factor

Secondly, the radiances are converted into reflectances $\rho^i(i, j)$:

$$\rho^i(i, j) = \frac{\pi L^i(i, j)}{I_0 \varepsilon \cos(\theta_s)} \quad (25)$$

Finally, we get a quantity ρ^{*t} that is a ground albedo at the instant t if the sky were clear:

$$\rho^{*t} = (\rho^t - \rho_{atm}^t) / T(\theta_s) T(\theta_v) \quad (26)$$

The ground albedo α_s is computed by taking the minimum value of the time series of ρ^{*t} .

The method described in this section relies on data collected by geosynchronous satellites. Meteosat-7 data were used in this study. The radiometer on-board the satellite measures radiation from the earth-atmosphere system in the shortwave range 0.40_1.10 μ m; so called visible channel and thermal infrared (10.50_12.50 μ m) regions of the electromagnetic spectrum. The sub satellite spatial resolution is 2.5x2.5 km² for the shortwave channel and 5x5 km² for the infrared channel. The data in the form of images containing non calibrated digital counts were acquired at 1200 UT in July 2000.

2.1.2. Statistical method

The first obvious effect of clouds is to reduce the total irradiation reaching the Earth, so the initial approach adopted here was to employ Heliosat method. This method as proposed by Cano et al (1986) is an estimation technique to refer the shortwave surface irradiance from satellite images [1]. The principle of this method is the construction of cloud index resulting from comparison of what is observed over that pixel [5]. The method is basically driven by the strong complementarity between the planetary albedo recorder by the satellite's radiometer and the surface shortwave radiant flux. The planetary albedo increases with growing atmospheric. For this propose, as a measure of cloud cover, a cloud index is derived from Meteosat visible counts for each pixel. This cloud index is defined as (Cano, 1986) [1]:

$$n = \frac{\rho - \rho_{min}}{\rho_{max} - \rho_{min}} \quad (27)$$

Where ρ , ρ_{min} , ρ_{max} are normalized values of the current, min and max satellite counts respectively current, clear and heavily overcast conditions. For a correct estimation of the incoming radiation on the total radiation reflected by an image element has been considered. Therefore, it is very important to normalize the satellite counts with respect to the zenith angle. A normalized count is computed as:

$$\rho = \frac{c_n - c_{n0}}{I} \quad (28)$$

Where c_n is the numerical count observed by the sensor, c_{n0} being what can be the sensor zero originally and I was assumed to be the surface global irradiance under a clear sky (Bourges, 1979).

Instead, here the extra terrestrial global irradiance I_{ext} is used. In this case, ρ is a measure for the planetary albedo. An analysis of a time series of the reflectance observed by the sensor is applied. The global solar radiation is computed using the global irradiance under clear sky, as given by the equation. The assessment of solar radiation from satellite images strongly depends on accuracy of the clear sky model that gives the value of I_{clear} . Therefore several clear sky models have been investigated by Rigollier and al (1999) [10]. In order to get a better accuracy for extreme cases, especially for very cloudy skies, Rigollier and Wald (1998) analysed several comparisons reported by European colleagues (Fontoynt et al., 1998) between ground measurements and estimates made through the use of the Heliosat method, and proposed a more suitable relationship:

$$\begin{cases} k = 1.2 & n \leq -0.2 \\ k = 1 - n & -0.2 \leq n \leq 0.8 \\ k = 2.0667 - 3.6667n + 1.6667n^2 & 0.8 \leq n \leq 1.1 \\ k = 0.05 & n \geq 1.1 \end{cases} \quad (29)$$

Finally, the global solar radiation I_g is computed using the global irradiation under clear sky I_{clear} , as given by the equation:

$$k = I_g / I_{clear} \quad (30)$$

Therefore several clear sky models have been investigated by Rigollier and al (1999) [11].

3. RESULTS AND DISCUSSION

For a series of days distributed along July 1998 and 2000, we evaluated at first the total radiation from the physical model. We took account of a standard atmosphere, which the values were used in the present study: water vapor content varies from 3 cm for the Saida's station and 3.5 cm for the Oran's station while the ozone content is assumed constant and equal to 0.34 Cm-atm. For the model of aerosols, we considered standard model SRA (standard radiation atmosphere WMO) for the aerosols of the maritim type for the Senia's station and continental type for the Saida's station.

In order to compare the satellite estimates of the global radiation with the direct measurements, data from the pyranometers network of the Algerian meteorological services (ONM) were acquired over the same period. The data were averaged over the period from 11 00 to 13 00 UT for comparisons with the satellite estimates at 12 00 UT in order to validate the method.

The comparison between the results obtained by satellite and measurements of pyranometer resulting from the weather stations proves to be delicate because of the difference of the two types of information. The pyranometer indeed gives a specific measurement integrated in time, whereas Meteosat gives a measurement instantaneous but integrated on a great surface ($2.5 \times 2.5 \text{ km}^2$). Integration in time and space presents a disadvantage for a comparison with measurements at the ground. For that we took into account the average value of the radiations calculated on a window of 3×3 pixels vicinity which represent the weather station. We notice that the lowest values of the total radiation which reaches the ground are generated by the cloud contaminated pixels. The lowest value observed is close to 52 W/m^2 . The average values are those at sea level of about 900 W/m^2 . The highest values are those in desert region during days of clear skies with the maximum value reached was 1056 W/m^2 on July 20th 2000 (Fig 1).

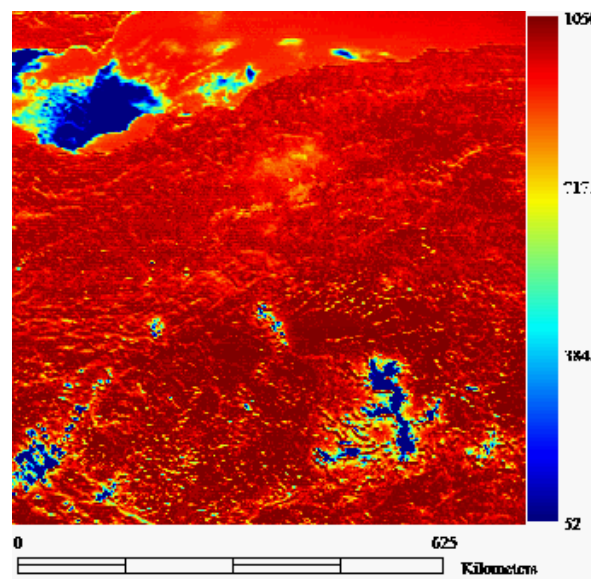


Fig. 1: Total radiation by unspecified sky of 20-07-2000 (W/m^2)

With some exceptions meadows, we note that the results of the solar radiation obtained by the physical method based on the spectral model seem very satisfactory and coincide well with the measured data resulting from the two stations, Senia and Saida. That is argued by the coefficients of correlation obtained by the comparison between the results obtained and measurements of the weather stations (Table 1). To validate the results obtained by the statistical method, we evaluated the total radiation by the statistical method at spectral base for a series of days distributed throughout July 1998 and 2000. The only parameter of entry for this method is the factor of turbidity of Linke. This last parameter was starting from a process of evaluation of atmospheric turbidity, based on the diffuse fraction.

With an aim of establishing the degree of validity of the results obtained from the two stations (Oran's and Saida's) we compared them with experimental measurements obtained from the two weather stations. A

fluctuation was recorded for the two days successively July 17th and 18th 2000 for certain pixels of which the error between the recorded value of the total radiation by the weather station and the computed value by the Heliosat method (Fig 2).

Table 1: Results of correlation obtained by the physical method

Days of analysis	Weather measuring site	Coefficient of correlation
3_20 July 1998	Senia's Station	0,9526105
3_20 July 2000	Senia's Station	0,72186185
3_20 July 2000	Saida's Station	0,95430889

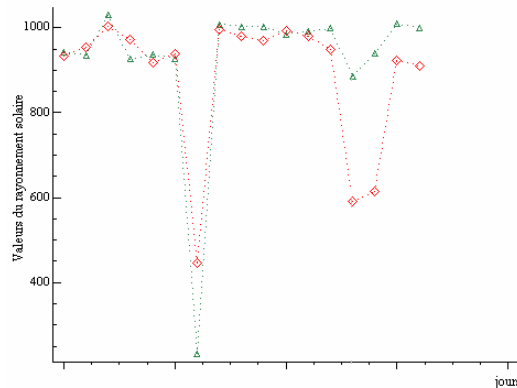


Fig. 2: Comparison between the measured values and the computed values of the 03 up to 20 July 2000 of the Senia Station by the Heliosat method

In order to define the cause, we worked out the calculation algorithm of the index of cloud. We noticed during the development of the algorithm that the pixel considered is classified as being a clear pixel, as well as the closest neighbours with a very weak index of cloud equalizes to 0.001, while the pixels close to the second order are contaminated where the index of cloud is higher or equal to 1.1. What we explain by the presence of the broken clouds where certain zones prove particularly delicate owing to the fact that one did not use the relations of space proximity (Fig 3). With the exception of the error quoted previously, we noticed that the results obtained are satisfactory. This is illustrated by the coefficients of correlation obtained (Table 2). The results obtained enabled us to conclude that this method is effective for the estimate of the total radiation on the ground, but it depends primarily on the measured data of atmospheric turbidity (Linke turbidity factor).

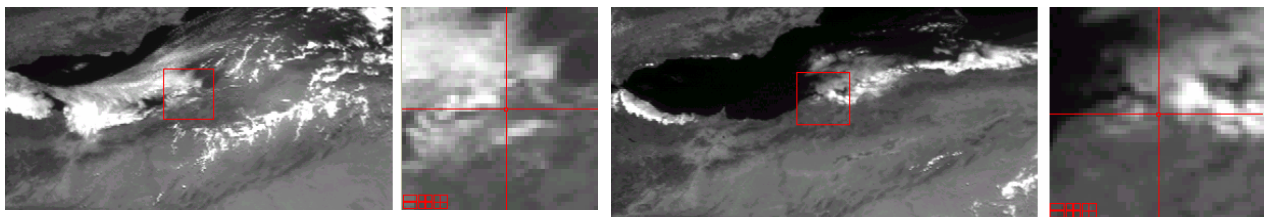


Fig. 3: Localization of the Senia station on the visible image for the two successive days: 17-07-2000 et 18-07-2000

Table 2 : Résultats de corrélation obtenus par la méthode statistique à base synthétique

Days of analysis	Weather measuring site	Coefficient of correlation
3_20 July 1998	Senia's Station	0,94727954
3_20 July 2000	Senia's Station	0,76377290
3_20 July 2000	Saida's Station	0,95633763

By carrying out a comparison between the results obtained by this physical method and those which are obtained by the statistical method we recorded a strong correlation. The coefficient of this correlation is equal to 0,97490697.

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4. REFERENCES

- [1] A. Hammer, D. Heichen, A. Westerhellweg, “*Derivation of daylight and solar irradiance data from satellite observations*”, 1997.
- [2] Christian Gueymard 1995. SMARTS2, “*A simple model of the atmospheric radiative transfer of Sunshine: Algorithms and performance assessment*”. FSEC-PF-270-95, Florida solar energy center, university of central Florida, USA (1995).
- [3] Christian Gueymard, “*A two band model for the calculations of clear sky solar irradiance, and photosynthetically active radiation at the earth’s surface.*”, Serres Bio-solaire Inc, Ham-Nord, GOP IAO Canada. Solar energy Vol. **43**, No. 5, pp.253-265, 1989.
- [4] C. G. Justus and M. V. Paris, “*A model for Solar Spectral Irradiance and Radiance at the Bottom and Top of the Cloudless*”, Atmosphere. School of geophysical Sciences, Georgia Institute of the technology, Atlanta, GA 30332. Journal of climate and applied Meteorology, Vol. **24**, No 3, 1985.
- [5] Daniel Cano. “*Etude de l’ennuage par analyse de Séquence d’images de satellite. Application à l’évaluation du rayonnement solaire global au sol*”. Thèse de doctorat, spatialité automatique et traitement du signal. Département Systèmes et communication. Ecole Nationale des télécommunications, France, 1982.
- [6] G.Dedieu, P.Y.Decschamps and Y.Kerr, “*Satellite estimation of solar Irradiance at the surface of the Earth and of surface albedo using a physical model applied to Meteosat data.*”, Laboratoire d’Etudes et de Recherches en Télédétection Spatiale, CNES-CNRS, 31055 Toulouse, France. Journal of climate and Applied Meteorology, Vol. **26**, No.1, January 1987.
- [7] I.Foyo_Moreno, J.Vida, F. J. Olmo, L. Alados-Arboledas, “*Estimating solar ultraviolet irradiance (290-385 nm) by means of the spectral parametric models*”: Spectral2 and Smarts 2, 2000.
- [8] Leckner, 1978 : Bo Leckner, “*The spectral distribution of solar radiation at the earth’s surface, Elements of the model*”, Dept. of Energy Conversion, Chalmers University of Technology, 402 20 Göteborg , Sweden. Solar energy, vol **20**, pp.143-150, 1978.
- [9] R. Bird, C.Riordon , “*Simple Solar Spectral Model for direct and diffuse irradiance on horizontal and tilted planes at the earth’s surface for cloudless atmospheres*”, 1986.
- [10] Rigollier and al, 1999: “*On the clear sky model of the Esra-European solar radiation atlas with respect to the Héliosat method.*”, Groupe Télédétection & modélisation, école des mines de Paris.[k]
- [11] Rigollier et al, 2000 : ENSMP / T&M (Rigollier C., Lefève M., Wald L)., “*Integration and exploitation of net worked solar radiation databases for environment monitoring*”,. Deliverable number :D3.2
- [12] J. W. Spencer,: “*Fourier Series Representation of the position of the sun.*”, Search, **2**, p.172, 1971