Bifurcation imperfection in double-diffusion-Soret convection in porous reservoirs due to lateral heating

A. Mansour 1, A. Amahmid 1, M. Hasnaoui 1* and M. Mamou 2

1 University Cadi Ayyad, Faculty of Sciences Semlalia, Department of Physics, UFR TMF, BP 2390 Marrakesh, Morocco
2 Institute for Aerospace Research, National Research Council, Ottawa, Ontario, Canada, K1A 0R6

Abstract - Thermo-diffusion and double-diffusion convection flows within multi-components enclosure subject to vertical and horizontal thermal gradients is investigated numerically and analytically. The focus of the present investigation is to determine the thresholds for oscillatory and stationary convection as function of the lateral heating factor and the Soret effect. Finite amplitude convection is also investigated and some universal correlations for the flow intensity and heat transfer characteristics are determined.

1. INTRODUCTION

The thermo-diffusion phenomenon has received a growing attention through the decades due to its practical interest in many industrial applications and also due to its presence in nature. More details concerning the applications of Soret effect in science and industry are given in a recent paper by Platten [1]. The author describes different techniques used to measure the Soret coefficient. Experimental results are provided for several systems, for both negative and positive Soret coefficients. Comparison between several laboratories is made for the same systems. Benchmark values of Soret coefficient are given for some organic liquid mixtures of interest in the oil industry. While studying experimentally the thermo-diffusion in a solution of sodium chloride contained in compact clay, Rosanne et al. [2] concluded that the mass transfer is enhanced by the thermal diffusion. Other than the experimental aspect, many theoretical investigations were also conducted to predict the Soret effect on the onset of convection flows in rectangular porous enclosures [3-5]. These studies on the Soret effect showed that this phenomenon may engender specific and unforeseeable behaviors in convective motions such as multiple steady/oscillatory states, subcritical flows, hysteresis behaviors, hopf bifurcations, reversal gradients of concentration and stabilization or destabilization of a system.

The present work is devoted to predict theoretically the onset of double-diffusive natural convection flows developed in a horizontal porous layer subject to uniform fluxes of heat and mass on its long sides. The imposed heat flux is more realistic in laboratory experiment and also justified by the fact that the temperature usually varies along the thermal active walls in practical situations. The study is focused on the special situation where the lateral perturbing heating flux is balanced by the horizontally induced Soret mass flux which allows an equilibrium state (motionless state) that becomes unstable under certain conditions. Using a finite element method, a linear stability analysis was performed to determine the threshold for any type of convective instabilities. The present thermo-diffusion problem was found to exhibit a rich variety of different bifurcation phenomena and complex unsteady flow patterns near criticality.

2. MATHEMATICAL FORMULATION

The flow configuration is a horizontal saturated porous enclosure filled with a binary mixture. To investigate thoroughly the effect of a lateral heating on the convective flow instabilities, a simple rectangular flow configuration is chosen for the ease of the mathematical and numerical modeling. The horizontal walls of the cavity are subject to uniform fluxes of heat, \( q' \), and
mass, \(j\), while the vertical ones are impermeable to mass but exposed to lateral heating by applying constant heat flux \(a_q\) where \(a\) is a constant real value. For simplicity, the porous medium is assumed well packed and isotropic where the inertia effects can be neglected based on low Reynolds flows assumption. Using the Darcy model and taking into account the Soret effect, the dimensionless governing equations are given in ref. [6].

3. ANALYTICAL SOLUTION

As a foundation of the correlations, an analytical solution was developed assuming steady-state convective flows. The solution is based on a parallel flow assumption, which assumes that, for an infinite aspect ratio enclosure, all the streamlines are parallel to the horizontal walls. The methodology and the solution are described in full details by Mansour et al. [6]. Thus, we recall here only the final solution, which is given by

\[
\Psi(y) = \Psi_0 \left(1 - 4y^2\right)
\]

\[
T(x, y) = C_T x - y - 4\Psi_0 C_T \left(\frac{y^3}{3} - \frac{y}{4}\right)
\]

\[
S(x, y) = C_S x + (M - 1)y - 4\Psi_0 \left(Le C_T + C_T \right) \left(\frac{y^3}{3} - \frac{y}{4}\right)
\]

where \(\Psi_0\) stands for the flow intensity or for the flow rate between any horizontal wall and the mid-height of the enclosure. This parameter can be quantified as

\[
\Psi_0 = \frac{R_T (C_T + NC_S)}{8}
\]

The expressions of the thermal and solutal horizontal gradients, \(C_T\) and \(C_S\), are given in Ref. [6].

The flow intensity is given by:

\[
Ck_1^4 + \left(\frac{k_2^2}{Le^2} - E\Psi_0^2\right)k_1^4 + \Psi_0^2 = 0
\]

\[
E = \frac{2}{Le^4} \left[\left|Le^2 + 1\right| - R_T^0 Le \left(N + Le\right)\right], \quad D = aR_T^0 Le \left(Le + 1\right), \quad C = \frac{4}{Le^2} \left[1 - R_T^0 Le \left(N - 1\right)\right]
\]

4. LINEAR STABILITY ANALYSIS OF THE REST STATE

For any set of values of the governing parameters, the rest state motionless solution is possible when the horizontally prescribed heat flux is balanced by the induced Soret mass flux within the porous layer i.e. for \(MN = 1\). By imposing infinitesimal perturbation to the rest state solution, \(f(x, y)e^{\mathbf{p}t}\), where the function \(f\) represents the spatial profile of the perturbation and \(p = p_t + ip_i\) its temporal growth rate, the discretized linear problem (resulting from the perturbed equations) leads to an eigenvalue problem which is solved with a finite element method and in which any of the governing parameters can be regarded as an eigenvalue and the spatial perturbation profiles as the eigenvectors. For the stationary/(oscillatory) convection, the Rayleigh number/(the growth \(p\)) is taken as the eigenvalue. The procedure for searching the onset of the oscillatory convection (overstabilities) for a given set of the governing parameters, \(Le, M, a,\) and \(A_r\), is performed by varying the Rayleigh number \(R_T\) while examining the real part of the
maximum eigenvalue. The critical Rayleigh number, $R_{TC}^{\text{over}}$, that characterizes the onset of overstability is determined when the real part of $p$ switches from negative to positive value (i.e. $p_r = 0$ and $p_i \neq 0$). The threshold, $R_{TC}^{\text{sup}}$, for the onset of stationary convection is obtained when $p = 0$.

5. RESULTS AND DISCUSSION

The stability diagram for the equilibrium state is displayed in figure 1 for $Le = 1.5$, $a = 0.2$ and infinite aspect ratio enclosure. The onset of different modes of convection depends strongly on the Soret parameter $M$. Within the range $[2/3, 2]$, subcritical convection exits and its threshold is delineated by $R_{TC}^{\text{sub}}$ curve. Above $R_{TC}^{\text{sub}}$, the motionless state becomes unstable to finite amplitude perturbation. For a given value of $M = 2/3$, the onset of subcritical convection is displayed in figure 2 by the turning saddle node point on the lower of the bifurcation diagram $R_{TC}^{\text{sub}}$. Then, the threshold $R_{TC}^{\text{sub}}$ is followed by the onset of a supercritical convection ($R_{TC}^{\text{sup}}$) on the right of the co-dimension 2 point (CD$_2$) and oscillatory convection characterized by $R_{TC}^{\text{over}}$ curve (overstability) on the left side. Beyond the thresholds, $R_{TC}^{\text{sup}}$, the motionless state becomes unstable to stationary infinitesimal perturbation. However, beyond the threshold $R_{TC}^{\text{over}}$, the perturbations are oscillatory. In general, the onset flow convection remains mono-cellular but it switches to multi-cellular flow on the right hand side of the transition point denoted by T in figure 1. As shown by the flow patterns on the diagram, the flow consists of tilted counter-rotating cells spanning the whole cavity length. Both the analytical and numerical stability analyses predict the existence of supercritical convection on the right hand side of the CD$_2$ point. On the left hand side, $R_{TC}^{\text{sup}}$ is found to exist according to the numerical solution, however according to the analytical solution which is based on the parallel flow assumption, the onset of supercritical convection does exist.

In figure 2, results are reported for $A_i = 1$ with $k_1 = 1.024$ and $k_2 = 3.425$ for the correlation. Analytical correlations were derived for the flow intensity $\Psi_0$ and fully numerical solutions were obtained with satisfactory agreement. In the presence of lateral heating $a = 0.2$, the lower side ($\psi_0 < 0$) of the diagram represents the natural clockwise solutions. On the upper side ($\psi_0 > 0$) are the anti-natural solutions. It can be seen that the natural solutions were more favored than the anti-natural ones as the formers display low thresholds of the onset of convection. Without lateral heating, $a = 0$, the natural and anti-natural solutions are identical with opposite flow rotation. The effect of the lateral heating enhances the flow intensity for the natural solution and acts to weaken the flow intensity for the anti-natural ones.

6. CONCLUSION

In simultaneous thermo-diffusion and double-diffusion convective flows in shallow enclosures, stability flow problem may arise when applying a lateral heating that balances the Soret induced mass flux. For this situation, different flow convective behaviors are possible such as the existence of subcritical, supercritical and overstable convective modes. Beyond the thresholds of instabilities, the lateral heating has the effect to enhance the natural flow and weaken the anti-natural one through its intensity and onset of convection delay.
Fig. 1: Stability diagram for $Le = 7$, $a = 0.5$ and $A_r = \infty$

Fig. 2: Bifurcation diagram for $Le = 5$, $a = 0$ and $0.2$, $M = 0.91$ and $A_r = 1$

**NOMENCLATURE**

- $A_r$ Aspect ratio
- $Le$ Lewis number
- $M$ Soret parameter
- $N$ Buoyancy ratio
- $T$ Dimensionless temperature
- $R_T$ Thermal Darcy-Rayleigh number
- $S$ Dimension less concentration

**Greek symbol**

- $\Psi$ Dimensionless stream function

**REFERENCES**


